Emergence of Populism under Ambiguity[*]

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Abstract

I construct a dynamic elections model with information asymmetries in which a representative voter who is ambiguity-averse chooses a policymaker among elites and non-elites in each period. Then, I investigate the effect of the uncertainty the voter faces about an elite's degree of bias on the emergence of populism. I show that an increase in *risk* and in *ambiguity* work in the opposite directions. An increase in risk makes populism less likely to arise so long as the reward and punishment mechanism to incentivize politicians is limited. By contrast, an increase in ambiguity makes populism more likely to arise. These results suggest that an increase in ambiguity rather than in risk is a crucial source of populism.

Keywords: Populism; Dynamic elections; Political agency; Ambiguity; Risk

JEL classification codes: D72; H11; D81; C73

1 Introduction

Populism has arisen often after a major change in society (e.g., in economic circumstances). For example, after the Great Depression, populism, especially extremism, emerged in several countries. Such a change often induces an increase in the uncertainty voters face. Therefore, an increase in uncertainty may be related to the emergence of populism. In this study, I investigate the relationship between the emergence of populism and an increase in the uncertainty voters face about politicians

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by distinguishing *ambiguity* and *risk*. In particular, by analyzing a dynamic incomplete information game with ambiguity-averse players, I show that an increase in risk and in ambiguity work in the opposite directions.

Anti-elitism is one of the aspects of populism.[1] Populism often arises as protest against politics by elites when ordinary citizens distrust them. Even if the elites have profound knowledge and ability, voters would vote for a non-elite when they think that the elites hurt voters' interests. This source of populism implies that populism is not necessarily a threat to democracy (Mudde and Kaltwasser 2012).[2] The concept of democracy requires that politics reflect the opinions of ordinary citizens. Thus, if politics by elites do not reflect the opinions of ordinary citizens, it is appropriate to elect a populist as the representative.

I develop a dynamic elections model that describes this aspect of populism. In the model, there are a representative voter (hereafter the voter), biased elites, and unbiased non-elites. A biased elite has sufficient ability to find out what is a good policy. However, her/his policy preference is biased from the voter's perspective. On the contrary, while an unbiased non-elite has only limited ability, her/his policy preference is the same as that of the voter since both are ordinary citizens. Here, a biased elite's degree of bias is drawn from a distribution, and its value is unobservable to the voter. In other words, the voter faces uncertainty about an elite's degree of bias. The voter elects the policymaker among the biased elites and unbiased non-elites in each period. Populism then emerges when the voter votes for an unbiased non-elite in spite of her/his limited ability.

The contribution of the present study is to analyze the effect of uncertainty based on this model. In reality, voters face uncertainty about an elite's degree of bias. I analyze how such uncertainty affects the emergence of populism. Here, as Knight (1921) points out, we should distinguish *ambiguity* (*Knightian uncertainty*), where even the probability distribution is unknown, from *risk*, where the probability distribution is known. Ellsberg's (1961) paradox shows the importance of this distinction. Thus, I analyze both cases by employing Choquet expected utility with a convex capacity (Schmeidler 1989). In other words, a special case of Maxmin expected utility (Gilboa and Schmei-

[1] The aspect I focus on is based on the following definition of populism which has been widely accepted in the field of political theory:

> I define populism as an ideology that considers society to be ultimately separated into two homogeneous and antagonistic groups, "the pure people" versus "the corrupt elite", and which argues that politics should be an expression of the volonté générale (general will) of the people. (Mudde 2004: 543)

[2] To understand this point, see the relationship between populism and direct democracy: "[T]he populist ideology shares the Rousseauian critique of representative government. [...] [P]opulism appeals to Rousseau's republican utopia of self-government, that is the very idea that citizens are able both to make the laws and execute them" (Mudde and Kaltwasser 2013: 503).

dler 1989) is employed: the voter has a set of priors about a biased elite's degree of bias and maximizes the payoff evaluated by the prior that gives the voter the minimum payoff.

Then, I show that the effect of an increase in uncertainty differs depending on which type of uncertainty is involved. When the voter has less confidence about the true distribution (i.e., the degree of ambiguity increases),[3] populism is more likely to arise. By contrast, when the variance in the distribution increases (i.e., the degree of risk increases), populism is less likely to arise so long as the reward and punishment mechanism to incentivize politicians is limited.[4] This is surprising because both risk and ambiguity are uncertainty, and thus both effects seem to be the same.

This result is obtained for the following reason. To begin with, the voter becomes reluctant to vote for a biased elite after an increase in both types of uncertainty because the voter is ambiguity-averse and risk-averse. However, this is not all. There is another effect that is different between risk and ambiguity. Under dynamic elections, the voter can replace the incumbent with a new one if s/he finds that the incumbent is highly biased. Therefore, the loss due to electing a highly biased elite is limited. This possibility of replacement creates the difference between risk and ambiguity.

In detail, an increase in risk (a mean-preserving spread) means that both the probability that an elite is not biased and the probability that an elite is highly biased increase. Here, the loss due to electing a highly biased elite is bounded thanks to the possibility of replacement. Thus, the benefit due to an increase in the probability that an elite is not biased dominates the loss due to an increase in the probability that an elite is highly biased. As a result, the expected payoff when the voter elects a biased elite increases with the degree of risk. Therefore, an increase in risk makes populism less likely to arise. To put it differently, the nature of dynamic elections (i.e., the possibility of replacement) makes democracy robust against populism even under high risk.

By contrast, this mechanism no longer works in the case of ambiguity. Roughly speaking, the voter's expected payoff when electing a biased elite is evaluated by using the prior that assigns the largest value to the probability that the degree of bias is quite high. After an increase in ambiguity, the set of candidates of the true distribution enlarges. As a result, the voter evaluates the payoff based on the prior that assigns a larger value to the probability that the degree of bias is quite high. Therefore, the voter's expected payoff when electing a biased elite decreases with the degree of ambiguity. By contrast, the payoff when electing an unbiased non-elite remains the same because there is no ambiguity about an unbiased non-elite's type. As a result, the voter becomes reluctant

[3]The employed notion of an increase in ambiguity is the expansion of the core of a convex capacity. However, this includes an increase in uncertainty aversion as well as ambiguity itself. These two cannot be separated in the framework of Choquet/Maxmin expected utility.

[4]Focusing on such a situation is meaningful because populism emerges only when controlling elites is hard.

to vote for a biased elite: an increase in ambiguity induces populism. Here, the probability that the degree of bias is low does not increase in contrast to the case of risk. Thus, the mechanism that the voter prefers more uncertainty thanks to the possibility of replacement does not work. In summary, an increase in ambiguity rather than risk is a significant source of populism.

The implications of this result are as follows. To begin with, voters would know the distribution of elites' preferences over a traditional policy issue because there have been a lot of opportunities for learning. Thus, voters face risk rather than ambiguity when the policy issue at the center of politics is a traditional one. In such a case, the distribution of an elite's degree of bias becomes more risky, as elites' policy preferences become more dispersed even among the elites whose direction of bias is the same (i.e., among the right-wing (resp. left-wing) elites).[5] When such a traditional but controversial issue arises at the center of politics, voters face more uncertainty about an elite's degree of bias. However, an increase in risk does not make populism more likely to arise. Thus, an increase in uncertainty due to the emergence of a controversial issue does not induce populism.

When do voters face an increase in ambiguity? When a new policy issue is at the center of politics, voters would not know even the distribution of elites' degree of bias because the information is too imprecise to be described by a single prior. In such a case, voters face ambiguity. Thus, the emergence of a new policy issue that has never been prioritized can be a source of populism. In addition, a major change in society often implies that a new policy issue arises. Thus, the result suggests that a significant change in society can be a source of populism because it makes the distribution of elites' policy preferences more ambiguous. This implication provides one explanation for the reality that populism is likely to arise after a large change in society. De Bromhead et al. (2012) and Funke et al. (2016) empirically show that populism is likely to emerge after economic crises.[6]

Lastly, I mention one difficulty in the analysis of a dynamic incomplete information game under ambiguity. In the case of ambiguity, dynamic consistency does not necessarily hold under simple updating rules corresponding to the Bayes rule. This fact makes the analysis difficult. Nonetheless, the present study succeeds in resolving this difficulty in a tractable manner. It is known that when rectangularity is satisfied, dynamic consistency holds in the framework of Maxmin expected utility (Epstein and Schneider 2003a). In the presented analysis, I restrict my attention to stationary equi-

[5]This point becomes clear after the formal definition of an elite's degree of bias is introduced.

[6]Another possible explanation is that a major change in society suggests that elites underperformed, with populism arising as a punishment for their failure. The explanation provided by the presented result is interesting compared with this alternative in that a significant change in society itself (not as a signal of the underperformance) can be a source of populism.

libria. Then, given others' stationary strategies, rectangularity holds for the voter in the derivation of the condition for the existence of an equilibrium in which populism does not arise. As a result, I do not have to care about dynamic inconsistency and updating rules.

The remainder of the paper proceeds as follows. Section 2 reviews the related literature. Section 3 describes the model. Section 4 derives the equilibrium. Section 5 defines an index to measure the likelihood of the emergence of populism. Section 6 analyzes the effect of an increase in uncertainty. Section 7 discusses an extension. Section 8 concludes.

2 Related Literature

■ **Political economics and ambiguity:** The present paper analyzes populism under ambiguity. Few studies deal with ambiguity in political economics (Berliant and Konishi 2005; Ghirardato and Katz 2006; Davidovitch and Ben-Haim 2010; Bade 2011; 2016; Baumann and Svec 2016; Ellis 2016; Yang 2016; Nakada, Nitzan, and Ui 2017), and no research thus far analyzed populism.

■ **Optimal stopping problem under ambiguity:** In the model, the voter's optimization problem is finally reduced to a variant of optimal stopping problems. Thus, outside of political economics, the present study is related to studies of optimal stopping problems under ambiguity. Nishimura and Ozaki (2004) analyze a one-sided labor search model, and show that the effect of an increase in ambiguity is different from that of an increase in risk in the model. Several studies have since derived a similar result in a different or general setting (e.g., Nishimura and Ozaki 2007; Miao and Wang 2011).[7] I employ Choquet expected utility as Nishimura and Ozaki (2004) do, and follow several of the assumptions introduced by them.

The present study has several features that are novel to the literature.[8] First, there is no analysis of political phenomena. The present study is the first application of an optimal stopping problem under ambiguity to the analysis of political phenomena.

Second, from a theoretical point of view, I adopt a dynamic game with information asymmetry, whereas the previous studies analyze one-person decision-making problems. The largest difficulty in the analysis of a dynamic situation under ambiguity is the dynamic inconsistency problem. One easy approach to guarantee dynamic consistency in a one-person decision-making problem is to assume an independent and indistinguishable distribution proposed by Epstein and Schneider

[7] Riedel (2009) analyzes the optimal stopping problem under ambiguity with discrete time generally, although he does not focus on the effect of an increase in ambiguity.

[8] There is also a technical difference from a typical optimal stopping problem. In the present paper, the voter chooses an action among three alternatives in each period, and nothing is irreversible, while there are only two alternatives of the action, and an irreversible choice exists in a typical optimal stopping problem.

(2003b).[9] Since information is completely uncovered under this assumption, rectangularity holds, and thus dynamic consistency is ensured (e.g., Nishimura and Ozaki 2004; 2007; Miao and Wang 2011). However, even under this assumption, rectangularity may fail in a dynamic incomplete information game because belief updating depends on players' strategies as well as an exogenous stochastic process. Thus, this approach cannot be applied to the analysis of a dynamic incomplete information game straightforwardly. Therefore, it is difficult to analyze a dynamic incomplete information game under ambiguity. Nonetheless, I succeed in solving it in a tractable manner by focusing on stationary equilibria. This is an important novelty of the present study.[10]

■ **Formal model of populism:** So far, I have reviewed the related studies in terms of applications of decision theory under ambiguity. Outside of them, there are several theoretical studies of populism, which are divided into two directions: signaling and pandering.[11] In the signaling literature, voters choose a politician who implements an extreme policy since this serves as a signal of the politician's good characteristics (e.g., Acemoglu, Egorov, and Sonin 2013).[12] In the pandering literature, voters have a belief about what is a good policy. To maintain a high reputation, politicians implement a policy consistent with voters' beliefs even though they know that it is harmful (e.g., Frisell 2009; Jennings 2011; Binswanger and Prufer 2012).[13]

The aspect of populism captured in these models is different from that captured in my model. In these models, populism is defined as a phenomenon that politicians choose policies hurting the interests of the majority of voters, despite with the majority of voters still support such politicians. Hence, it is not described as a phenomenon that voters vote for a non-elite who has only limited ability because they distrust elites. As a result, the aspect of populism being harmful to voters is highlighted.

■ **Dynamic elections with adverse selection:** Lastly, the model has a common feature with dynamic elections models with adverse selection. Duggan (2000) develops an infinite horizon model in which voters choose the policymaker among citizen candidates. In his model, candidates' policy preferences are unobservable, and the voters decide whether to replace the incumbent with a new

[9] Another approach is to employ a complicated updating rule under which dynamic consistency holds.

[10] Boyarchenko and Levendorskiĭ (2012) analyze a game-theoretic situation (a preemption game). However, their game is without information asymmetry. In such a setting, the belief is only based on the exogenous stochastic process. In this sense, their model is close to a one-person decision making problem in terms of belief formation.

[11] Other related studies include those of populist economic policies (e.g., Mejía and Posada 2007; Leon 2014) and policy divergence in electoral competition (e.g., Boleslavsky and Cotton 2015).

[12] The strand of the literature begins with the study of Kartik and McAfee (2007), which does not mention populism.

[13] The studies above relate pandering to populism. Pioneering studies in the literature are those by Harrington (1993), Canes-Wrone, Herron, and Shotts (2001), and Maskin and Tirole (2004), although they do not refer to populism.

candidate, whose policy preference is drawn from a distribution.[14] Then, he characterizes a simple equilibrium where politicians are divided into three types: (i) centrists whose policy preferences are close to that of a median voter and who implement their own ideal policy and can be reelected, (ii) moderates who adopt the most extreme policy that allows them to be reelected, and (iii) extremists whose policy preferences are far from that of a median voter and who implement their own ideal policy and cannot be reelected.[15]

Several differences from the literature exist. First, no study analyzes the effect of an increase in uncertainty.[16] Second, the focus differs. In the literature, all the candidates are citizen candidates (i.e., ordinary citizens). By contrast, I distinguish a non-elite candidate who is an ordinary citizen from an elite candidate who is not an ordinary citizen, and analyze populism in the framework of dynamic elections.

3 The Model

3.1 Setting

The model has an infinite horizon: $t = 0, 1, ...$ The society consists of ordinary citizens and elites. The ordinary citizens are homogeneous in terms of policy preferences, and they have the power to choose who conducts policymaking. Due to the homogeneity, it suffices to focus on a representative voter (hereafter, the voter). Each player has a policy preference on a unidimensional policy space $(-\infty, \infty)$. In period t, the voter chooses who s/he will elect as the policymaker. After that, the elected politician chooses policy x_t. This sequential game is infinitely repeated.

3.1.1 Voter

Let the voter's ideal policy in period t be $\hat{x}_t$. The desirable policy varies depending on the external circumstances. Thus, $\hat{x}_t$ is a stochastic variable that differs over time. It is independently drawn from a probability distribution F whose density function is f. F is assumed to be symmetric: $f(a) = f(-a)$ for any $a \in [0, \infty)$. Since the voter is unfamiliar with policy issues, the value of $\hat{x}_t$ is unknown to the voter, whereas F is known.

The voter's payoff in each period is a linear loss function: $-|x_t - \hat{x}_t|$. In other words, the voter

[14] Banks and Sundaram (1993) also analyze a similar situation, but their focus is effort choice by a politician.

[15] Subsequent studies include Banks and Duggan (2008), Bernhard et al. (2009), Bernhard, Câmera and Squintani (2011), and Câmera and Bernhard (2015).

[16] Câmera and Bernhard (2015) analyze a change in the distribution about policy preferences. However, they consider the effect of a decrease in polarization as opposed to an increase in uncertainty.

is risk-neutral. Note that Berinsky and Lewis (2007) show empirically that a voter's loss function is almost linear. Section 7 discusses risk-aversion. As mentioned in Section 3.1.3, the value of $-|x_t - \hat{x}_t|$ is unobservable to the voter until the game has ended so long as monitoring is not successful.

3.1.2 Politicians

There are two types of politicians in each period: a biased elite and an unbiased non-elite. The biased elite observes the value of $\hat{x}_t$ perfectly after s/he is elected as the policymaker. In this sense, s/he has the ability to act for the voter. However, his/her policy preference is different from that of the voter. Hence, s/he may implement a policy different from $\hat{x}_t$.

Each biased elite's policy preference is biased compared with that of the voter:

1. Right-biased elite: the ideal policy is $\hat{x}_t^r(\beta) = \hat{x}_t + \beta$, where $\beta \in [0, \bar{\beta}]$.

2. Left-biased elite: the ideal policy is $\hat{x}_t^l(\beta) = \hat{x}_t - \beta$, where $\beta \in [0, \bar{\beta}]$.

Here, $\bar{\beta} \in (0, \infty)$. When a biased elite is elected as the policymaker in period t, s/he receives a payoff from policymaking and the office-seeking motivation. The payoff due to the policy mismatch in period t is $-|x_t - \hat{x}_t^i(\beta)|$, where $i = r, l$. In addition, during the period in the office, s/he receives $\rho \in (0, \infty)$ which represents the office-seeking motivation. In summary, during the period in office, a biased elite receives $-|x_t - \hat{x}_t^i(\beta)| + \rho$. On the contrary, when a biased elite is not in the office, her/his payoff is zero since the mental cost due to the policy mismatch would be small during this period compared with during the period in office.[17]

Next, an unbiased non-elite can observe the value of $\hat{x}_t$ with probability $\phi \in [0, 1)$ after being elected.[18] Note that this is independent of the value of $\hat{x}_t$. Thus, an unbiased non-elite has only limited ability. Let $\hat{x}_t^o \in (-\infty, \infty) \cup \emptyset$ be the observed value of $\hat{x}_t$. $\hat{x}_t^o = \emptyset$ represents that an unbiased non-elite cannot observe $\hat{x}_t$, and $\hat{x}_t^o \in (-\infty, \infty)$ represents that s/he observes that $\hat{x}_t = \hat{x}_t^o$. The advantage of an unbiased non-elite is literally the unbiasedness of her/his policy preference. The

[17] A biased elite is different from an ordinary citizen. Therefore, her/his economic utility would be unaffected by government policies compared with that of an ordinary citizen. For example, social security system would not affect the economic utility of a person who has high ability and obtains high income. In this sense, a biased elite's disutility from the policy mismatch is due to the mental cost rather than the economic or physical cost. During the period in office, the mental cost would be large compared with that when not in office since the policy mismatch is implemented by her/himself during the period in office. The setting above captures this reality.

[18] An unbiased non-elite's ability of finding the state of the world $\hat{x}_t$ is assumed to be lower than that of a biased elite. An alternative setting is that the ability for policy implementation is different. Both elites and non-elites observe $\hat{x}_t$. However, to achieve the policy goal, the policy maker must choose the details of policies appropriately. In particular, suppose that an unbiased non-elite has limited knowledge so that s/he knows how to implement policy x only with probability ϕ, while a biased elite knows how to implement it. Then, the same result is obtained though Lemma 1 slightly changes.

payoff due to the policy mismatch in period t is the same as that of the voter: $-|x_t - \hat{x}_t|$. During the period in office, an unbiased non-elite receives $-|x_t - \hat{x}_t| + \rho$, while her/his payoff is zero during the period not in office.[19]

Lastly, a politician who loses an election will never stand again as a candidate.

3.1.3 Information Asymmetries and Voter's Decision

At the beginning of period t, there are four (three) politicians when $t \geq 1$ ($t = 0$): (i) the incumbent who was elected as the policymaker in period $t-1$, and (ii) the alternative candidates consisting of a right-biased elite, a left-biased elite, and an unbiased non-elite (when $t = 0$, the incumbent does not exist). Here, the degrees of bias of right and left-biased elites, who are alternative candidates in period t (hereafter new biased elites), are drawn from the same distribution, and this is common knowledge. The voter elects one of them as the policymaker in each period.[20]

The voter can distinguish between a right-biased elite, a left-biased elite, and an unbiased non-elite. However, the voter is uncertain of biased elites' degrees of bias. This is the first information asymmetry (hidden information). In addition, the voter cannot observe the implemented policy x_t and the desirable policy $\hat{x}_t$. As a result, the voter cannot observe the implemented policy mismatch $|x_t - \hat{x}_t|$ in principle. This is the second information asymmetry (hidden action).

However, these information asymmetries are resolved directly or indirectly through monitoring. The voter observes the implemented policy mismatch $|x_t - \hat{x}_t|$ with probability $q \in (\underline{q}, \bar{q})$, where $0 < \underline{q} < \bar{q} \leq 1$, at the end of each period. Note that this is independent over time. This can be regarded as monitoring by the mass media since media outlets gather news and report on the incumbent's performance with some probability in reality. Thanks to this monitoring, both information asymmetries are partially resolved. Since $|x_t - \hat{x}_t|$ is observed after monitoring, the second information asymmetry is resolved. In addition, after $|x_t - \hat{x}_t|$ is observed, the voter may be able to infer the incumbent's degree of bias. Thus, the first information asymmetry is also resolved. Whether monitoring is successful is observable to politicians as well as the voter.

[19] Once the interpretation discussed in the previous footnote is taken into account, an unbiased non-elite's payoff during the period not in office may depend on the policy mismatch since s/he is an ordinary citizen in contrast to a biased elite. In such a setting, the almost same result holds since Lemma 1 does not change.

[20] An alternative setting is that there are both a large election and preliminary elections. Suppose that there are two parties: right-wing and left-wing. The right-wing (left-wing) party endorses a right (left) biased elite. The party to which the incumbent belongs endorses the incumbent automatically. The opposite party endorses a biased elite drawn from a distribution. Thus, there are the incumbent and a biased elite who are endorsed by each party in a large election. In addition, an unbiased non-elite exists as a candidate in a large election. In summary, there are three candidates in each large election. This setting is basically the same as that of Bernhard et al. (2009) except for the existence of an unbiased non-elite. In this setting, the number of candidates is reduced to three without changing any result.

Owing to these information asymmetries, the voter faces a trade-off between a biased elite and an unbiased non-elite. A biased elite has sufficient ability, but s/he may choose a policy far from $\hat{x}_t$. On the contrary, an unbiased non-elite has only limited ability, but there is no conflict of interests. I say that "populism emerges" when the voter chooses an unbiased non-elite as the policymaker.

3.1.4 Timing

To consider a situation where the implemented policy mismatch is unobservable with some probability, the voter's payoff due to the policy mismatch should not be realized in each period. To this end, suppose that the game ends at the end of each period independently with probability $1 - \delta$, where $\delta \in (0, 1)$. When the game ends, the voter's payoff is realized. The innate discount rate is zero, and thus the discount factor is δ.

The timing of each stage game is as follows:

1. Nature draws the values of β of new biased elites from a distribution.

2. The voter votes for one of the candidate.

3. Nature draws the value of $\hat{x}_t$ from distribution F. Then, the elected politician observes $\hat{x}_t$ with probability one if s/he is a biased elite, and with probability ϕ if s/he is an unbiased non-elite.

4. The elected politician chooses policy x_t.

5. The voter observes $|x_t - \hat{x}_t|$ with probability q.

3.2 Ambiguity about Politicians' Types

The voter does not know a biased elite's degree of bias. I allow a situation where even the distribution of β is unknown. In this subsection, I describe this ambiguity.

Let $(B, \mathcal{F}_B)$ be a measurable space, where $B = [0, \bar{\beta}]$, and $\mathcal{F}_B$ is the Borel $\sigma-$algebra on B. Each element $\beta \in B$ represents the degree of bias of a biased elite. For any $t \geq 0$, I construct the t-dimensional product measurable space $(B^t, {\mathcal{F}_B}^t)$ (let ${\mathcal{F}_B}^0 \equiv \{\emptyset, B^\infty\}$) and embed it into the infinite-dimensional product measurable space $(B^\infty, \mathcal{F}_B^\infty)$.

3.2.1 Beliefs

I need to consider two types of the voter's beliefs: (i) belief about the degree of bias of a new biased elite, and (ii) belief about the degree of bias of the incumbent biased elite. Suppose that the voter elected a biased elite as the policymaker in the previous period. Then, in the next election, the voter must decide whether to reelect this incumbent. Here, the voter should take into account the degree of bias of (i) new biased elites, and (ii) the incumbent. Thus, both must be specified.

Belief formation is based on history. At the end of period t, the voter observes $s_t \in S_t = D_t \times A_t$. First, $D_t = \{[0, \infty), \emptyset\}$ with its generic element d_t, and this represents information about the implemented policy mismatch in period t. $d_t = d \in [0, \infty)$ means that the voter finds out that $|x_t - \hat{x}_t| = d$. Further, $d_t = \emptyset$ means that the voter does not find out the value of $|x_t - \hat{x}_t|$. Second, $A_t = \{0, r, l, u\}$ when $t \geq 1$, and $A_t = \{r, l, u\}$ when $t = 0$. This represents a voting action in period t. 0 represents reelecting the incumbent, r (l) represents electing a new right-biased (left-biased) elite, and u represents electing a new unbiased non-elite. The history, which has been observed by the voter until the beginning of period $t \geq 1$, is $s^{t-1} = (s_0, s_1, ..., s_{t-1}) \in S^{t-1} = \Pi_{\tau=0}^{\tau=t-1} S_\tau$. The null history S^{-1} is set to be $\{\emptyset\}$.

Consider (i). Let $\theta_t : S^{t-1} \times \mathcal{F}_B \to [0, 1]$, and call this a capacity kernel.[21] For any $A \in \mathcal{F}_B$, $\theta_{t,s^{t-1}}(A)$ represents a capacity such that the degree of bias of a new biased elite in period t is in A, given history s^{t-1}. This construction allows the voter to update θ_0 based on the past history.

I assume the following stochastic process. When θ_0 is additive (i.e., in the case of risk), β follows an independent and identical distribution over time. When θ_0 is non-additive (i.e., in the case of ambiguity), β follows an independent and indistinguishable distribution over time (Epstein and Schneider 2003b).[22] Then, $\theta_{t,s^{t-1}}$ is independent of t and s^{t-1}, and so θ_t is time-homogeneous. This assumption has been widely used (e.g., Epstein and Wang 1994; 1995; Nishimura and Ozaki 2004; 2007; Miao and Wang 2011). Without notational abuse, let $\theta_{t,s^{t-1}}$ be θ for all $t \geq 0$ and $s^{t-1} \in S^{t-1}$.

Next, consider (ii). Although the focus is the incumbent biased elite, I consider the belief about the incumbent, which includes the case where the incumbent is an unbiased non-elite as well as the case where the incumbent is a biased elite, since it simplifies the notation. Denote its capacity kernel by $\theta'_t : S^{t-1} \times \mathcal{F}_B \to [0, 1]$ for any $t \geq 1$. This is updated based on the Naive Bayes rule.[23]

Although how θ'_t is updated depends on the strategies in general, I specify the belief in the following two cases in which the belief does not depend on the strategies. Let $\underline{\tau}(t)$ be the period when

[21] Although the concept of a kernel is usually used to describe a Markov process, here I call the above a capacity kernel.

[22] Suppose that the data-generating mechanism is independent and identical. When the voter knows the distribution, s/he does not update her/his belief since there is nothing left to learn. By contrast, when the voter does not know the distribution, the voter updates her/his belief. Thus, under ambiguity, the IID assumption does not induce a time homogeneous capacity. However, there could be a situation in which the capacity is independent of t and s^{t-1}. This is the independent and indistinguishable distribution. The voter thinks that the data-generating process differs over time, but s/he does not understand how it differs. Thus, the voter learns nothing, but the belief is the same over time.

[23] The Naive Bayes rule is $\theta(A|B) = \frac{\theta(A \cap B)}{\theta(B)}$ for any $A, B \in \mathcal{F}_B$. Other rules are possible so long as the belief in Section 4.2.3 is obtained.

the incumbent at the beginning of period $t \geq 1$ was elected as the policymaker for the first time. Then, $d^t_{\underline{\tau}(t)} = D^{t-\underline{\tau}(t)+1}$ represents the history of the observed $|x_t - \hat{x}_t|$ implemented by the incumbent until period t. When $d^t_{\underline{\tau}(t)} = \emptyset^{t-\underline{\tau}(t)+1}$ and $a_{\underline{\tau}(t)} \in \{r, l\}$ (i.e., when the voter has never observed the policy implemented by the incumbent biased elite), the voter should not update her/his belief; $\theta'_{t,s^{t-1}} = \theta$. Moreover, when $a_{\underline{\tau}(t)} = u$ (i.e., when the incumbent is an unbiased non-elite), the voter knows that the incumbent's degree of bias is zero, and thus $\theta'_{t,s^{t-1}}(\{0\}) = 1$.

Lastly, θ is assumed to be convex, continuous, and full-support on $[0, \bar{\beta}]$. Continuity guarantees the Fubini property (see Nishimura and Ozaki 2004). In addition, all the probability distribution functions in core(θ) are assumed to be continuous.[24] Note that a probability charge in the core of a continuous capacity is countably additive and hence a probability measure. As a result, the distinction between a probability charge and a probability measure does not matter. See Appendix A for the details about these assumptions.

3.2.2 Payoffs and Equilibrium Concept

Define the voter's payoff by the iterated (i.e., recursive) Maxmin payoff whose set of probability measures in each period is equivalent to the core of the aforementioned capacity in each period. Thus, the voter's payoff is the iterated Choquet expected payoff based on the aforementioned capacity kernel. This equivalence comes from the following relationship: let u be bounded and measurable, and v be a convex and continuous capacity. Then

$$\int u(\beta)dv = \min\left\{\int u(\beta)dG \middle| G \in \text{core}(v)\right\}.^{25} \tag{1}$$

Note that the integral in the left-hand side is Choquet integral. Choquet expected utility with a convex capacity is equivalent to Maxmin expected utility whose set of priors is the core of the capacity. Here, a situation is reduced to decision making under risk when the capacity is additive (or equivalently when its core is a singleton).

For the equilibrium concept, I use the following one that is a natural analogue to Perfect Bayesian Equilibrium. I restrict my attention to pure strategies.

Definition 1 *The strategies and the belief system* $(\theta, \{\theta'_t\}_{t=1}^{\infty})$ *constitute an equilibrium if*

[24]Under this assumption, the existence of a solution to the Bellman equation is easily ensured (Lemma 8). Although it may be possible to guarantee the existence without continuity, I employ this assumption since the main purpose is to analyze the emergence of populism, and complicated technical issues are outside of the scope of this study.

[25]The minimum is attained since u is assumed to be bounded and measurable, and also core(θ) is weak* compact by the Alaoglu theorem. For the ease of notation, G represents not only a probability measure itself but also its probability distribution function in the following sections.

1. *the strategies are sequentially rational for any $t \geq 0$, and*
2. *the belief system is consistent with the strategies in the sense that the belief is updated based on the Naive Bayes rule (i.e., the full Bayesian updating rule in Maxmin expected utility) so long as it is possible.*

In the case of ambiguity, whether the definition above is appropriate is not clear.[26] This is due to a possibility of dynamic inconsistency. The recursive/iterated Maxmin payoff from period t is not necessarily equivalent to the non-iterated Maxmin payoff evaluated by using only the core of the capacity in period t: the law of iterated expectation does not necessarily hold. To put it differently, when the latter payoff is employed, dynamic inconsistency can arise.

However, in the candidates of equilibrium on which I focus (i.e., a kind of stationary equilibrium), dynamic consistency trivially holds since rectangularity, which guarantees dynamic consistency (Epstein and Schneider 2003a), is satisfied as I show later. Thus, both payoffs are equivalent for the voter given others' stationary strategies. As a result, the distinction between them does not matter, and hence I can employ the recursive payoff and use dynamic programming methods.[27]

To see this rigorously later, define the non-iterated payoff. To this end, fix each player's strategy. Since politicians' strategies only depend on the public history and β in equilibria on which I focus later, suppose that politicians' strategies only depend on them. Let p_t be a stochastic kernel for the belief about a new biased elite's degree of bias in period t, and p'_t be a stochastic kernel for the belief about the incumbent biased elite's degree of bias in period t. For any $t \geq 1$ and s^{t-1} under which the voter elects a left (right) biased elite in period t, further, let $r^l_{t,s^{t-1}}$ ($r^r_{t,s^{t-1}}$) be an objective stochastic kernel about the value of d_t. In particular, $r^l_{t,s^{t-1},\beta}$ ($r^r_{t,s^{t-1},\beta}$) is a probability measure about the value of d_t when the policymaker in period t is a left-biased (right-biased) elite whose degree of bias is β, and the history is s^{t-1}. Similarly, for any $t \geq 1$ and s^{t-1} under which the voter elects an unbiased non-elite in period t, let $r'_{t,s^{t-1}}$ be an objective probability measure about the value of d_t.

By using p_τ ($\tau \geq t$), p'_t, $r^l_{\tau,s^{\tau-1}}$ ($\tau \geq t$), $r^r_{\tau,s^{\tau-1}}$ ($\tau \geq t$), and $r'_{\tau,s^{\tau-1}}$ ($\tau \geq t$), one can construct a probability measure p^t about the sequence of the implemented policy mismatch, given the history s^{t-1}. Here, for each τ, $p_{\tau,s^{\tau-1}} \in \text{core}(\theta)$, and $p'_{t,s^{t-1}} \in \text{core}(\theta'_{t,s^{t-1}})$ for any $s^{t-1} \in S^{t-1}$. Denote the set of p^t given s^{t-1}

[26] In general, the appropriate equilibrium concept is more complicated since the complicated updating rule that guarantees dynamic consistency should be used (see Hanany, Klibanoff, and Mukerji 2016).

[27] The condition under which the payoff evaluated by using the initial capacity is equivalent to the payoff calculated recursively is still unclear (see Yoo (1991) and Dominiak (2013)) although that in the framework of Maxmin expected utility is provided by Epstein and Schneider (2003a). The verification above is based on the framework of Maxmin expected utility theory. For this reason, Maxmin expected utility is initially employed to define the voter's payoff.

by $\mathcal{P}_{t,s^{t-1}}$. Then, the voter's non-iterated Maxmin payoff from period t conditional on s^{t-1} is

$$\inf_{p^t \in \mathcal{P}_{t,s^{t-1}}} EP_t(p^t, s^{t-1}),$$

where $EP_t(p^t, s^{t-1})$ is the expected payoff from period t conditional on s^{t-1} using p^t. Here, minimum takes only once, and the payoff is not calculated recursively. Later, I show that even if the payoff in Definition 2 is employed instead of the recursive payoff, the same result holds.[28]

4 Equilibrium

4.1 Equilibrium Refinement

Owing to the information asymmetries and the repeated game structure, there may be a number of equilibria, which complicates the analysis. To avoid such complications, the literature has put restrictions on equilibrium strategies and beliefs, and focused on symmetric stationary equilibrium (Duggan 2000; Banks and Duggan 2008; Bernhard et al. 2009; Bernhard, Câmera and Squintani 2011; Câmera and Bernhard 2015).[29] I follow the same convention, although the details are different.

To begin with, define $\tau^*(t)$ as follows: when $t \geq 1$

$$\tau^*(t) \equiv \begin{cases} \emptyset \ \ ((\forall \tau \in \{\underline{\tau}(t), ..., t-1\}) \ \ d_\tau = \emptyset.) \\ \max\left\{\tau \in \{\underline{\tau}(t), ..., t-1\} \middle| d_\tau \neq \emptyset\right\} \ \ \text{(otherwise)} \end{cases},$$

and when $t = 0$, $\tau^*(t) \equiv \emptyset$. Here, $\tau^*(t)$ is the latest period such that the policy mismatch, implemented by the incumbent in period t, was observed. $\tau^*(t) = \emptyset$ represents that the voter has never observed the policy mismatch implemented by the incumbent. Thus, $d_{\tau^*(t)}$ is the policy mismatch observed in period $\tau^*(t)$, namely the latest observed policy mismatch implemented by the incumbent. Note that in the original definition of d_t, $d_{\tau^*(t)}$ is not well defined when $\tau^*(t) = \emptyset$. Thus, when $\tau^*(t) = \emptyset$, $d_{\tau^*(t)}$ is set to be $\emptyset$ without notational abuse. By using these notations, I introduce one class of voting strategy, satisfying stationarity, and assume that the voter's equilibrium strategy belongs to this class. Since voters have bounded rationality in real world, focusing on this simple class is realistic.

[28]In this framework, the strategies and belief system constitute an equilibrium if the strategies are sequentially rational, and core(θ_t') is updated by using the full Bayesian updating rule so long as it is possible. Although I use the full Bayesian updating rule, other updating rules are possible so long as the belief in Section 4.2.3 is obtained.

[29]Duggan (2014), who shows the Folk theorem, is the exception. However, even he points out "the application of dynamic electoral models will rely on equilibrium refinements (e.g., the common restriction to stationary equilibria)." Moreover, the present model is more complicated than his model.

Assumption 1 *The voter's equilibrium strategy must satisfy the following. When the incumbent is a biased elite, for any history, the voter decides whether to voter for the incumbent, a new biased elite, or a new unbiased non-elite, based on the same rule r: the decision in period t only depends on* $d_{\tau^*(t)}$*, i.e.,* $r : [0, \infty) \cup \emptyset \to \{0, 1, u\}$*, where* 0 *represents reelection,* 1 *represents voting for a new biased elite, and u represents voting for a new unbiased non-elite.*

Next, politicians' equilibrium strategies are assumed to satisfy stationarity and symmetry. A policymaker chooses $|x_t - \hat{x}_t|$ by choosing x_t. In the following assumption, I consider the choice of policy mismatch $|x_t - \hat{x}_t|$.

Assumption 2 *(i) A biased elite's equilibrium strategy must satisfy the following.* $|x_t - \hat{x}_t|$ *only depends on* β *and* $\hat{x}_t$*, and this decision rule is the same over time, unless s/he has never observed the deviation from the equilibrium strategies since s/he became the policymaker*[30]*. (ii) In addition, an unbiased non-elite's equilibrium strategy must satisfy the following.* $|x_t - \hat{x}_t|$ *only depends on* $\hat{x}_t^o$*, and this decision rule is the same over time, unless s/he has never observed the deviation from the equilibrium strategies since s/he became the policymaker.*

The voter must infer the incumbent's degree of bias only through the observed $|x_t - \hat{x}_t|$. Without stationarity, inference becomes hard. Note further that the above imposes symmetry, too, since a right-biased elite and a left-biased elite whose degrees of bias are the same choose the same policy mismatch.

Assumptions 1 and 2 imply stationary equilibria. In this sense, these two restrictions can be regarded as equilibrium refinement.

Lastly, consider the voter's belief about the off-equilibrium paths. To eliminate equilibria whose off-equilibrium belief is not plausible, I impose one restriction on the voter's belief.

Assumption 3 *The voter's belief that constitutes an equilibrium must satisfy the following. Suppose that the incumbent is a biased elite. When the voter observes d such that no elite chooses it given the history, the voter believes that the incumbent elite's degree of bias* β *is* $\min\{d, \bar{\beta}\}$*.*

This is about the belief on the incumbent's degree of bias when the voter observes an off-equilibrium policy mismatch. If the voter observes off-equilibrium policy mismatch d, which is smaller than or equal to $\bar{\beta}$, the voter should believe that the incumbent's degree of bias is d. This is verified by assuming that with very small fraction, there is an extremely self-interested biased elite who always

[30] Suppose that the elite is chosen as the policymaker in period t. This restriction means that the elite has never observed any deviation since stage 3 in period t.

implements her/his own ideal policy. When d is larger than $\bar{\beta}$, there is no biased elite whose bias is d. In that case, this assumption requires that the voter believes that the degree of bias is $\bar{\beta}$, which is closest to d.

4.2 Benchmark: Payoff when an Unbiased Non-Elite is the Policymaker

To begin with, derive the voter's expected payoff when electing an unbiased non-elite as the policymaker in every period. In each period, an unbiased non-elite observes the value of $\hat{x}_t$ with probability ϕ. In this case, the unbiased non-elite implements policy $\hat{x}_t$. On the contrary, with probability $1-\phi$, the unbiased non-elite cannot observe the value of $\hat{x}_t$. In this case, s/he chooses a policy that is the solution of the following problem to minimize the expected loss due to the policy mismatch: $\min_{x_t} \int_{-\infty}^{\infty} |x_t - \hat{x}_t| dF$. It is well-known that the solution of this problem is the median of x_t. Thus, the unbiased non-elite chooses 0 as x_t since the median is zero from the symmetry of F. Therefore, when the value of $\hat{x}_t$ is unobservable, the voter's expected payoff in period t is $\int_{-\infty}^{\infty} -|\hat{x}_t| dF = -2\int_0^{\infty} \hat{x}_t dF$.

In summary, I obtain the following lemma. All the proofs are contained in Appendix B.

Lemma 1 *The voter's expected payoff when electing an unbiased non-elite as the policymaker in every period is*

$$-\frac{2(1-\phi)}{1-\delta}\int_0^{\infty} \hat{x}_t dF. \tag{2}$$

4.3 Equilibrium in which Populism does not Emerge

I derive the necessary and sufficient condition for the existence of an equilibrium where populism does not arise in that the voter never votes for an unbiased non-elite on the equilibrium path.

Here, F is a symmetric distribution, and β of a new right-biased elite and a new left-biased elite are drawn from the same distribution. Thus, a right-biased elite and a left-biased elite are totally indifferent for the voter. Therefore, I do not distinguish whether a biased elite is left or right.[31]

4.3.1 Preliminaries

To begin with, I obtain the following lemma straightforwardly.

[31]One may think that electing a left-biased elite or a right-biased elite would affect a biased elite's strategy since the incumbent right-biased elite has less incentive to deviate if the voter elects a not right but left-biased elite after her/his deviation. This is the essence of "party competition effect" discussed by Bernhardt et al. (2009). If this is the case, electing a left-biased elite or a right-biased elite affects the voter's payoff. However, such a possibility does not exist in my model since the incumbent's payoff is zero after s/he is replaced with a new politician.

Lemma 2 *Suppose that there is an equilibrium. Denote the voter's payoff from period* 0*, when the voter elects a biased elite as the policymaker in period 0, and the players follow the equilibrium strategies after the period 0 election, by* $\tilde{V}$*. Then,*

1. *the voter's expected payoff from period* $t \geq 1$ *when the voter votes for a new biased elite in period* t*, and the players follow the equilibrium strategies after the period* t *election, and*

2. *the voter's expected payoff from period* $t \geq 1$ *when in period* t *the voter reelects the incumbent biased elite whose implemented policy mismatch has never been observed, and the players follow the equilibrium strategies after the period* t *election*

are also $\tilde{V}$*.*

Here, I impose the following assumption.

Assumption 4 *The following inequality holds:*

$$\max\left\{\int_0^{\bar{\beta}} \beta dG \middle| G \in \mathrm{core}(\theta)\right\} > 2(1-\phi)\int_0^{\infty} \hat{x}_t dF.$$

Suppose that every biased elite chooses her/his ideal policy when s/he is elected as the policymaker, and the voter cannot replace her/him with another candidate. This is the worst scenario. Then, the expected payoff when the voter continues to elect a biased elite as the policymaker is

$$\frac{1}{1-\delta}\min\left\{-\int_0^{\bar{\beta}} \beta dG \middle| G \in \mathrm{core}(\theta)\right\}. \tag{3}$$

If it is optimal for the voter to vote for a biased elite even under this worst scenario, the analysis is meaningless since populism never arises. Thus, suppose that (3) < (2). This is Assumption 4.

Denote $\tilde{V}$ in the equilibrium where the voter never votes for an unbiased non-elite on the equilibrium path by V. Then, I obtain the several lemmas.

Lemma 3 *Suppose that the voter never votes for an unbiased non-elite on the equilibrium path. Given the history, if the voter's equilibrium strategy is to replace the incumbent biased elite when* $d_{\tau^*(t)} = d > 0$*, the voter believes that the incumbent's degree of bias* $\beta = \min\{d, \bar{\beta}\}$ *when* $d_{\tau^*(t)} = d > 0$*.*

Lemma 4 *Suppose that the voter never votes for an unbiased non-elite on the equilibrium path. Denote the voter's payoff from period* $t \geq 0$ *when the voter votes for a new unbiased non-elite in period* t*, and the players follow the equilibrium strategies after the period* t *election by* V_u*. Then,* $V_u \leq V$*.*

Hereafter, I suppose that there is an equilibrium in which the voter never votes for an unbiased non-elite on the equilibrium path. Then, I derive the necessary and sufficient condition for the existence of such an equilibrium (Theorem 1). To this end, I derive the strategies in this equilibrium (Section 4.3.2). Then, based on the strategies, I obtain the voter's payoff in this equilibrium (Sections 4.3.3- 4.3.5). Lastly, I examine the condition for the existence of this equilibrium (Section 4.3.6).

4.3.2 Strategies

To begin with, derive the voter's equilibrium strategy. On this issue, I obtain the following lemma.

Lemma 5 *Every equilibrium outcome, in which the voter never votes for an unbiased non-elite on the equilibrium path, (if it exists), can be constructed by the voting strategy r having the following property: there is $\beta^* \in [0, \bar{\beta})$ such that the voter reelects the incumbent biased elite if $d_{\tau^*(t)} \leq \beta^*$ and does not reelect the incumbent biased elite if $d_{\tau^*(t)} > \beta^*$.*

Therefore, without loss of generality, I focus on the threshold voting strategy such that the voter reelects the incumbent biased elite if $d_{\tau^*(t)} \leq \beta^*$ and does not reelect her/him if $d_{\tau^*(t)} > \beta^*$.

Next, derive a biased elite's strategy given the voter's strategy. Who has an incentive to implement a policy such that $|x_t - \hat{x}_t| \leq \beta^*$? Obviously, a biased elite, whose $\beta \leq \beta^*$, has an incentive to do so since s/he can be reelected after choosing his/her ideal policy $\hat{x}_t + (-)\beta$. In addition, even a biased elite, whose β is larger than β^*, may have an incentive to implement policy mismatch β^* for reelection since the benefit of reelection ρ exists. This incentive exists if and only if

$$\frac{\rho - (\beta - \beta^*)}{1 - \delta} \geq \rho + \delta(1 - q)\frac{\rho - (\beta - \beta^*)}{1 - \delta} \Leftrightarrow \beta \leq \beta^{**} \equiv \beta^* + \frac{q\delta\rho}{1 - (1 - q)\delta}.$$

Since all the probability measures contained in core(θ) do not have an atom at the point of β^{**} from the assumption on θ, whether a biased elite, whose degree of bias is β^{**}, chooses the compromised policy mismatch β^* does not affect the equilibrium outcome. Thus, I assume that such a biased elite chooses policy mismatch β^*. Let β^{***} be $\min\left\{\beta^{**}, \bar{\beta}\right\}$. From the discussion above, a biased elite whose $\beta \in (\beta^*, \beta^{***}]$ will implement policy mismatch β^*. In summary, I obtain the following lemma.

Lemma 6 *A right-biased (left-biased) elite whose degree of bias is β follows the strategy below:*

$$x_t = \begin{cases} \hat{x}_t + (-)\beta \;\; (\beta \in [0, \beta^*]) \\ \hat{x}_t + (-)\beta^* \;\; (\beta \in (\beta^*, \beta^{***}]) \\ \hat{x}_t + (-)\beta \;\; (\beta \in (\beta^{***}, \bar{\beta}]) \end{cases}.$$

The discussion above does not depend on whether the voter reelects the incumbent biased elite when $d_{\tau^*(t)} = \emptyset$. In either case, a biased elite's strategy is described by Lemma 6. In addition, reelecting the incumbent and electing a new biased elite as the policymaker are indifferent for the voter since both payoffs are V from Lemma 2. Therefore, whether the voter reelects the incumbent biased elite when $d_{\tau^*(t)} = \emptyset$ does not affect the equilibrium outcome. For this reason, I do not specify whether the voter reelects the incumbent biased elite when $d_{\tau^*(t)} = \emptyset$.

These derived strategies of the voter and a biased elite share a common feature with those derived in the literature on dynamic elections with information asymmetry. Lastly, the following lemma is obtained.

Lemma 7 $-\frac{\beta^*}{1-\delta} = V$ *holds.*

4.3.3 Beliefs

The next step is to derive the voter's belief. I specify the belief when the incumbent is a biased elite as follows:

1. When $d_{\tau^*(t)} = \emptyset$, $\theta'_{t,s^{t-1}} = \theta$.
2. When $d_{\tau^*(t)} = \beta \in [0, \bar{\beta}] \setminus \{\beta^*\}$, $\theta'_{t,s^{t-1}}(\{\beta\}) = 1$.
3. When $d_{\tau^*(t)} = \beta^*$, $\theta'_{t,s^{t-1}}([\beta^*, \beta^{***}]) = 1$.
4. When $d_{\tau^*(t)} = \beta \in (\bar{\beta}, \infty)$, $\theta'_{t,s^{t-1}}(\{\bar{\beta}\}) = 1$.

Consider the belief on the equilibrium path. 1 must hold since there is no information for updating. Further, if the voter has ever observed $\beta \in [0, \beta^*)$ or $\beta \in (\beta^{***}, \bar{\beta}]$ since the incumbent won the seat, the voter must believe that the incumbent's degree of bias is β from the politician's strategy. 2 includes this. In addition, if the voter has ever observed β^*, the voter must believe that the incumbent's degree of bias is in $[\beta^*, \beta^{***}]$ from the politician's strategy. 3 includes this. Here, I specify off equilibrium belief arbitrary. Although other off equilibrium beliefs exist, these do not affect the determination of β^* and β^{***}.

There is one remark on the belief specified in 3. In 3, I specify only $\theta'_{t,s^{t-1}}([\beta^*, \beta^{***}])$ and do not specify $\theta'_{t,s^{t-1}}(A)$ for $A \subset [\beta^*, \beta^{***}]$. This is because which β among $[\beta^*, \beta^{***}]$ is the incumbent's degree of bias is payoff irrelevant for the voter. Since the voter receives the same payoff whatever value the incumbent's degree of bias takes among $[\beta^*, \beta^{***}]$, the voter only uses $\theta'_{t,s^{t-1}}([\beta^*, \beta^{***}])$ when calculating her/his payoff.

In this belief formation, the payoff relevant information on the incumbent's degree of bias is perfectly revealed or completely not revealed.[32] It is well-known that rectangularity holds in such a case (Epstein and Schneider 2003a). Thus, given this belief and a biased elite's strategy, rectangularity holds. From this observation, the following lemma is obtained.

Lemma 8 *There is an equilibrium in which the voter never votes for an unbiased non-elite on the equilibrium path when the payoff is iterated one if and only if there is such an equilibrium when the payoff is non-iterated one.*

In general, the iterated payoff is not necessarily equivalent to the non-iterated payoff. However, regardless of whichever payoff is employed, the condition for the existence of an equilibrium in which populism does not arise is the same. This is the verification for using the recursive payoff.

4.3.4 Bellman Equation

From the discussion on strategies and beliefs, I obtain the following Bellman equation:

$$V = \min \Bigg\{ \Bigg[-\int_0^{\beta^*} \beta dG - \int_{\beta^*}^{\beta^{***}} \beta^* dG - \int_{\beta^{***}}^{\bar{\beta}} \beta dG \Bigg] \\ + \delta(1-q)V + \delta q \Bigg[-\frac{1}{1-\delta} \int_0^{\beta^*} \beta dG - \frac{1}{1-\delta} \int_{\beta^*}^{\beta^{***}} \beta^* dG + \int_{\beta^{***}}^{\bar{\beta}} V dG \Bigg] \Bigg| G \in \text{core}(\theta) \Bigg\}. \tag{4}$$

Without loss of generality, focus on period 0. The expected payoff in period 0 by electing a biased elite as the policymaker is the first term. If the elected elite's degree of bias is $\beta \notin [\beta^*, \beta^{***}]$, s/he just chooses her/his own ideal policy from Lemma 6. Thus, the loss for the voter is β. When the elected elite's degree of bias is $\beta \in [\beta^*, \beta^{***}]$, s/he chooses policy mismatch β^* for reelection. Thus, the loss for the voter is β^*. The second and third terms represent the expected payoff from period 1. With probability $1-q$, the voter cannot observe the implemented policy mismatch. In this case, the voter reelects the incumbent biased elite or elects a new biased elite as the policymaker. Then, the expected payoff from period 1 is V from Lemma 2. This is the second term. On the contrary, with probability q, the voter observes the implemented policy mismatch. This is the third term. When the observed policy mismatch is smaller than or equal to β^*, the voter believes that the incumbent biased elite will continue to choose the same policy mismatch, and the voter reelects her/him from

[32] When the voter has not observed the policy mismatch implemented by the incumbent, any information is not revealed. When the voter observed $\beta \in [0, \beta^*)$ or $\beta \in (\beta^{***}, \bar{\beta}]$, the incumbent's degree of bias is completely revealed. When the voter observed $\beta = \beta^*$, the voter finds that the policy mismatch implemented by the incumbent is β^* forever, and hence payoff relevant information is revealed.

Lemmas 5 and 6. When the observed policy mismatch is larger than β^*, the voter replaces the incumbent with a new biased elite. In this case, the expected payoff is V from Lemma 2.

Note that in general, $\int_a^c f(\beta)d\theta = \int_a^b f_1(\beta)d\theta + \int_b^c f_2(\beta)d\theta$ does not hold when $f(\beta) = f_1(\beta)$ if $\beta \in [a, b)$, and $f(\beta) = f_2(\beta)$ if $\beta \in (b, c]$ since the integral is the Choquet integral. On the contrary, $\int_a^c f(\beta)d\theta = \min\left\{\int_a^b f_1(\beta)dG + \int_b^c f_2(\beta)dG \middle| G \in \mathrm{core}(\theta)\right\}$ holds from relationship (1). Therefore, the expression by Maximin expected utility is employed.

4.3.5 Voter's Equilibrium Payoff

Since $V = -\frac{\beta^*}{1-\delta}$ holds from Lemma 7, equation (4) is equivalent to

$$-\frac{\beta^*}{1-\delta} = \min\left\{-\int_0^{\beta^*} \beta dG - \int_{\beta^*}^{\beta^{***}} \beta^* dG - \int_{\beta^{***}}^{\bar{\beta}} \beta dG + \delta q\left[-\frac{1}{1-\delta}\int_0^{\beta^*} \beta dG - \frac{1}{1-\delta}\int_{\beta^*}^{\bar{\beta}} \beta^* dG\right] \middle| G \in \mathrm{core}(\theta)\right\}$$
$$- \delta(1-q)\frac{\beta^*}{1-\delta}. \tag{5}$$

By solving equation (5), β^* and V are obtained. If β^* is uniquely determined from this equation, V is also uniquely determined.

Hereafter, I show the existence of a unique β^* that is the solution to equation (5). Denote

$$h(\tilde{\beta}) = -(1-\delta(1-q))\frac{\tilde{\beta}}{1-\delta} - \min\left\{-\int_0^{\tilde{\beta}} \beta dG - \int_{\tilde{\beta}}^{\min\left\{\tilde{\beta}+\frac{q\delta\rho}{1-(1-q)\delta}, \bar{\beta}\right\}} \tilde{\beta} dG - \int_{\min\left\{\tilde{\beta}+\frac{q\delta\rho}{1-(1-q)\delta}, \bar{\beta}\right\}}^{\bar{\beta}} \beta dG \right.$$
$$\left. +\delta q\left[-\frac{1}{1-\delta}\int_0^{\tilde{\beta}} \beta dG - \frac{1}{1-\delta}\int_{\tilde{\beta}}^{\bar{\beta}} \tilde{\beta} dG\right] \middle| G \in \mathrm{core}(\theta)\right\}. \tag{6}$$

This is the left-hand side minus the right-hand side of equation (5). When $h(\tilde{\beta}) = 0$, $\tilde{\beta}$ is equal to β^* that satisfies equation (5). I prove several lemmas about the properties of $h(\tilde{\beta})$.

Lemma 9 *$h(\tilde{\beta})$ is a decreasing function of $\tilde{\beta} \in \left[0, \bar{\beta} - \frac{\delta q\rho}{1-(1-q)\delta}\right]$, and $h(\tilde{\beta}) < 0$ holds for $\tilde{\beta} \in \left[\bar{\beta} - \frac{\delta q\rho}{1-(1-q)\delta}, \bar{\beta}\right]$.*

From the second part of this lemma, $\beta^* \in \left[0, \bar{\beta} - \frac{\delta q\rho}{1-(1-q)\delta}\right)$. Moreover, since $h(\tilde{\beta})$ is monotonically decreasing with $\tilde{\beta} \in \left[0, \bar{\beta} - \frac{\delta q\rho}{1-(1-q)\delta}\right]$, there is only a unique solution if β^* such that $h(\tilde{\beta}) = 0$ exists.

The next lemma is about the continuity of $h(\tilde{\beta})$.

Lemma 10 *$I(\tilde{\beta}) = \min\left\{J(\tilde{\beta}, G)\middle| G \in \mathrm{core}(\theta)\right\}$ is continuous with respect to $\tilde{\beta} \in (0, \bar{\beta})$ if J is continuous function.*

When core(θ) is a singleton, $h(\tilde{\beta})$ is continuous. However, its continuity is not necessarily obvious when core(θ) is not a singleton. By using this lemma, I have the continuity of $h(\tilde{\beta})$. The first term of h is obviously continuous. Thus, I focus on the second term. For the second term of $h(\tilde{\beta})$,

$$J(\tilde{\beta}, G) = -\int_0^{\tilde{\beta}} \beta dG - \int_{\tilde{\beta}}^{\min\left\{\tilde{\beta}+\frac{q\delta\rho}{1-(1-q)\delta}, \bar{\beta}\right\}} \tilde{\beta} dG - \int_{\min\left\{\tilde{\beta}+\frac{q\delta\rho}{1-(1-q)\delta}, \bar{\beta}\right\}}^{\bar{\beta}} \beta dG + \delta q \left[-\frac{1}{1-\delta}\int_0^{\tilde{\beta}} \beta dG - \frac{1}{1-\delta}\int_{\tilde{\beta}}^{\bar{\beta}} \tilde{\beta} dG\right].$$

Since $G \in$ core(θ) is a continuous distribution function from the assumption, $J(\tilde{\beta}, G)$ is obviously continuous. Thus, the second term is also continuous from Lemma 10. In summary, $h(\tilde{\beta})$ is continuous. This property is used to show that a solution to $h(\tilde{\beta}) = 0$ exists.

By using Lemmas 9 and 10, I obtain the following result for the existence of a unique β^*.

Lemma 11 *There always exists a unique $\beta^* \in \left(0, \bar{\beta} - \frac{\delta q \rho}{1-(1-q)\delta}\right)$ that satisfies equation (5).*

In summary, the value of β^* and thus the value of V are uniquely determined.

4.3.6 Equilibrium

Finally, I obtain the condition for the existence of an equilibrium in which populism does not emerge.

Theorem 1 *There is an equilibrium, in which the voter never votes for an unbiased non-elite on the equilibrium path on the equilibrium path, if and only if for β^* that satisfies equation (5),*

$$\beta^* \le \bar{\beta}^* \equiv 2(1-\phi)\int_0^{\infty} \hat{x}_t dF \tag{7}$$

holds.

In any equilibria, where the voter never votes for an unbiased non-elite on the equilibrium path, the voter's strategy must be described by Lemma 5, and a biased elite's strategy must be described by Lemma 6. Further, β^* that characterizes this possible equilibrium is uniquely determined by solving $h(\tilde{\beta}) = 0$. It also means that the value of V is uniquely determined.

If the value of V is higher than or equal to (2), it is optimal for the voter to vote for a biased elite in every period. Otherwise, the voter has a strong incentive to vote for an unbiased non-elite and thus populism emerges. Hence, $V \geq$ (2) is the necessary and sufficient condition for the existence of an equilibrium in which the voter votes for a biased elite in every period. Condition (7) is obtained by rewriting $V \geq$(2).

In this sense, condition (7) determines whether populism arises. When the value of β^* exceeds $\bar{\beta}^*$, populism always arises. Otherwise, there is an equilibrium in which populism does not arise.

5 Monitoring Ability

In this section, I examine the effect of monitoring ability q on the emergence of populism.

To begin with, I obtain the following lemma.

Lemma 12 *β^* is decreasing with q.*

Therefore, as monitoring ability q increases, V becomes larger. This is because the two agency problems are mitigated by a high monitoring ability. The first one is the moral hazard problem. The voter controls the incumbent biased elite by replacing the incumbent if the observed policy mismatch is larger than β^*. Hence, biased elites, whose bias is between β^* and β^{**}, choose policy mismatch β^*. The higher q is, the larger β^{**} is since the incumbent biased elite has less incentive to deviate. This is the first positive effect of an increase in q on the voter's payoff. The second one is the adverse selection problem. The voter may choose a highly biased elite as the policymaker. When the monitoring ability is high, the voter can detect the incumbent biased elite, whose β is high, with high probability. This means that even if the voter elects a highly biased elite as the policymaker, s/he can replace this elite with a new biased elite with high probability. Thus, the concern about electing a highly biased elite becomes smaller as q increases. This is the second positive effect. Through these two paths, the value of voting for a biased elite increases with q.

Define q, which is nonnegative and where the solution to equation (5) is $\bar{\beta}^*$, by $\underline{q}^*$. Here, $\underline{q}^*$ is not necessarily in $(\underline{q}, \bar{q})$. Thus, let $\underline{q}^{**} \equiv \min\{\max\{\underline{q}, \underline{q}^*\}, \bar{q}\}$. I obtain the following proposition.

Proposition 1 *There is a unique $\underline{q}^{**}$. (i) When $\underline{q}^{**} = \bar{q}$, condition (7) does not hold. (ii) When $\underline{q}^{**} \in (\underline{q}, \bar{q})$, condition (7) holds if and only if $q \in (\underline{q}^*, \bar{q})$. (iii) When $\underline{q}^{**} = \underline{q}$, condition (7) holds.*

Thus, the monitoring ability must be high to prevent populism from arising. Since q represents the monitoring ability of the mass media, it suggests that the distrust of the mass media induces populism. This is consistent with the current situation where populism arises and trust in the mass media is undermined. Indeed, the trust of the pubic in the mass media has been decreasing over time (Ladd 2011; Pew 2011).

Note that $\underline{q}^{**}$ represents the least requirement of the monitoring ability to prevent populism. Thus, a decrease in $\underline{q}^{**}$ means that populism is less likely to arise. In the next section, $\underline{q}^{**}$ is used as an index to measure the likelihood of the emergence of populism.

6 An Increase in Uncertainty

I examine how an increase in the uncertainty about a biased elite's degree of bias affects the emergence of populism by examining the effect on $\underline{q}^{**}$.

6.1 Effect of an Increase in Risk

I analyze the effect of an increase in uncertainty in the sense of *risk*. For this purpose, I employ a standard notion that measures the degree of risk: *mean-preserving spread*.

In the case of risk, θ is additive. Denote the additive capacity (i.e., probability measure) by G. I compare two probability distributions G_1 and G_2, and assume that both G_1 and G_2 are differentiable. The density function of each distribution is denoted by g_1 and g_2 respectively.

Lemma 13 *Suppose that probability distribution G_1 is a mean-preserving spread of probability distribution G_2. Then, for any $\tilde{\beta} \in [0, \bar{\beta}]$,*

$$\int_0^{\tilde{\beta}} G_1(\beta) d\beta \geq \int_0^{\tilde{\beta}} G_2(\beta) d\beta. \tag{8}$$

Since G_1 is the mean-preserving spread of G_2, G_2 second order stochastically dominates G_1. The property in Lemma 13 is the definition of the second order stochastically dominance.

The next lemma provides us with the alternative representation of $h(\tilde{\beta})$. Since the cumulative distribution function is assumed to be differentiable, this expression is possible.

Lemma 14 *$h(\tilde{\beta})$ can be rewritten as follows:*

$$h(\tilde{\beta}) = -\tilde{\beta} + \int_0^{\bar{\beta}} \beta dG + \int_{\tilde{\beta}}^{\min\left\{\tilde{\beta} + \frac{q\delta\rho}{1-(1-q)\delta}, \bar{\beta}\right\}} (\beta - \tilde{\beta}) dG - \frac{\delta q}{1-\delta} \int_0^{\tilde{\beta}} G(\beta) d\beta. \tag{9}$$

By using the preceding lemmas, I derive the proposition about the effect of an increase in risk.

Proposition 2 *Suppose that probability distribution G_1 is a mean-preserving spread of probability distribution G_2.*

(a) Suppose that inequality (8) holds with a strong inequality when $\tilde{\beta} = \bar{\beta}^*$. Then, there is $\bar{\rho} > 0$ such that for $\rho \in (0, \bar{\rho})$, $\underline{q}^{**}(G_1) \leq \underline{q}^{**}(G_2)$.

(b) Suppose that both G_1 and G_2 are symmetric and unimodal distributions, and there is $\hat{\beta} \in (0, \bar{\beta})$ such that $g_1(\beta) > g_2(\beta)$ if $\beta \in (\hat{\beta}, \bar{\beta}/2)$, and $g_1(\beta) \leq g_2(\beta)$ if $\beta \in [0, \hat{\beta}]$.[33] Then, $\underline{q}^{**}(G_1) \leq \underline{q}^{**}(G_2)$ holds if (i) $\hat{\beta} \leq \bar{\beta}^* \leq \bar{\beta}^* + \frac{\underline{q}^*(G_1)\delta\rho}{1-(1-\underline{q}^*(G_1))\delta} \leq \bar{\beta} - \hat{\beta}$, or (ii) $\hat{\beta} \geq \bar{\beta}^*$, and $\bar{\beta} - \hat{\beta} \leq \bar{\beta}^* + \frac{\underline{q}^*(G_1)\delta\rho}{1-(1-\underline{q}^*(G_1))\delta} \leq \bar{\beta} - \bar{\beta}^*$ is satisfied.

[33] This trivially holds when both are truncated normal distributions.

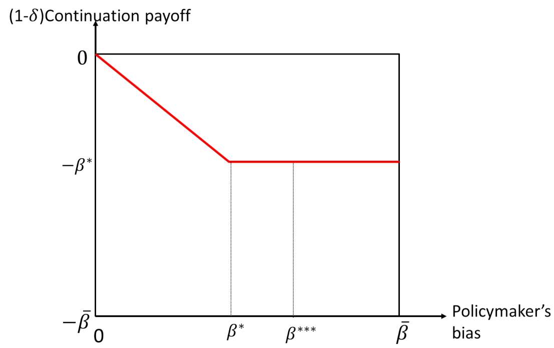

Figure 1: Continuation Payoff

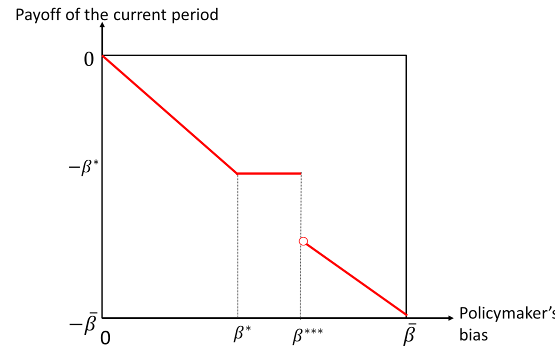

Figure 2: Payoff of the Current Period

It seems that the more uncertain an elite's bias, the more reluctant the voter is to vote for the elite. However, Proposition 2 (a) argues that so long as uncertainty is risk, this is not the case when the reward and punishment mechanism to incentivize biased elites is limited (i.e., ρ is small). Rather, as the degree of risk increases, populism is less likely to arise. Since whether populism emerges matters only when it is difficult for the voter to control a biased elite (i.e., when the voter faces the severe agency problem), the result when ρ is small is meaningful.

The mechanism behind this result is as follows. Focus on the expected continuation payoff from period $t+1$ that is evaluated in period t given that the voter observes the policy mismatch. This payoff is given by

$$-\frac{1}{1-\delta}\int_0^{\beta^*}\beta dG - \frac{1}{1-\delta}\int_{\beta^*}^{\beta^{***}}\beta^* dG + \int_{\beta^{***}}^{\bar{\beta}} V dG = -\frac{1}{1-\delta}\int_0^{\beta^*}\beta dG - \frac{1}{1-\delta}\int_{\beta^*}^{\bar{\beta}}\beta^* dG$$

from equation (4) and Lemma 6. Figure 1 shows the continuation payoff as a function of the policymaker in period t's degree of bias. This function is convex. The convexity is induced by the possibility of replacement. The voter can replace the incumbent with a new one if the voter finds that the incumbent is highly biased. Thus, even if the voter elects a highly biased elite whose degree of bias is $\beta \in (\beta^{***}, \bar{\beta}]$, the voter can obtain V as the continuation payoff. Note that V is the expected payoff when the voter elects a new biased elite. As a result, convexity is obtained. Therefore, the voter behaves as if s/he were a risk-lover. Thus, the mean-preserving spread increases the value of electing a biased elite. Hence, the possibility of replacement — thanks to the nature of dynamic elections — makes populism less likely to arise after an increase in risk.

Thus far, I have focused on the effect on the continuation payoff. An increase in risk also affects the payoff of the current period. Figure 2 describes the payoff function of the current period when

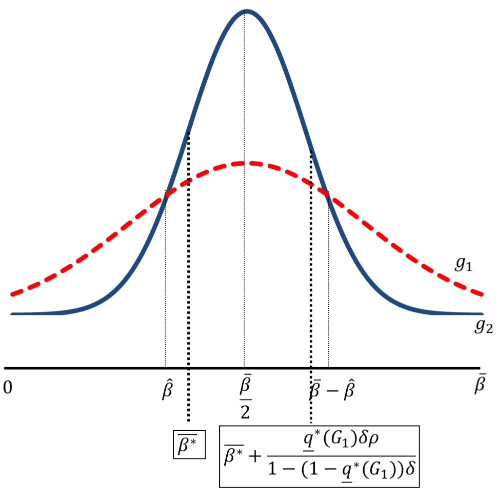

Figure 3: Condition (i) in Proposition 2 (b)

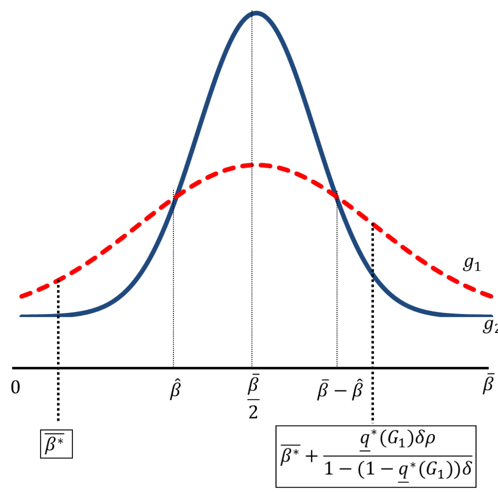

Figure 4: Condition (ii) in Proposition 2 (b)

the voter elects an elite whose degree of bias is β. In contrast to Figure 1, this payoff function is not convex. Thus, the effect of the mean-preserving spread on the expected payoff of the current period is unclear, and could be negative. After the mean-preserving spread, the probability that $\beta \in [\beta^*, \beta^{**}]$ may decrease while the probability that $\beta \in (\beta^{**}, \bar{\beta}]$ may increase. Thus, the expected payoff of the current period may be smaller after the mean-preserving spread, depending on the distribution functions. However, when ρ is small, the probability of $\beta \in [\beta^*, \beta^{**}]$ is low since it becomes hard to control a policy implemented by a biased elite. As a result, the effect due to a decrease in the probability of $\beta \in [\beta^*, \beta^{**}]$ is negligible. Therefore, when ρ is small, the positive effect on the continuation payoff always dominates the possible negative effect on the payoff of the current period. For this reason, a sufficiently small ρ is necessary in Proposition 2 (a).

Lastly, it should be emphasized that sufficiently small ρ is only a sufficient condition. Suppose that an increase in risk increases the current payoff when voting for a biased elite. Then, the increase in risk always makes populism less likely to arise. When does such a situation arise? Proposition 2 (b) presents a simple case, where an increase in risk makes populism less likely to arise even if ρ is not necessarily small. In the proposition, I consider symmetric distributions. Then, I show that when the values of $\bar{\beta}^*$ and $\bar{\beta}^* + \frac{\underline{q}^*(G_1)\delta\rho}{1-(1-\underline{q}^*(G_1))\delta}$ satisfy a property that can be seen in Figures 3 and 4, populism is less likely to arise as the degree of risk increases.

6.2 Effect of an Increase in Ambiguity

I showed that an increase in risk makes populism less likely to arise. Given this, it would seem that that an increase in ambiguity also has a similar effect. However, an increase in ambiguity makes populism more likely to arise.

To begin with, define an increase in ambiguity.

Definition 2 *θ_1 is more ambiguous than θ_2 if for any $A \in \mathcal{F}_B$, $\theta_1(A) \leq \theta_2(A)$ holds.*

This definition is also employed in the literature (Nishimura and Ozaki 2004; 2007; Miao and Wang 2011). Since both θ_1 and θ_2 are convex, this is equivalent to core(θ_1) $\supseteq$ core(θ_2). Remember relationship (1). The expansion of the core of a capacity means that the set of priors enlarges. Thus, this definition of an increase in ambiguity means that the set of candidates of the true distribution expands. Note that this definition includes an increase in uncertainty aversion as well as that in ambiguity itself.[34] Although it should be noted as a limitation, disentangling an increase in ambiguity from that in uncertainty aversion has never been succeessful in the framework of Choquet/ Maxmin expected utility.[35]

By using this definition, I obtain the proposition on the effect of an increase in ambiguity.

Proposition 3 *Suppose that θ_1 is more ambiguous than θ_2. Then, $\underline{q}^{**}(\theta_1) \geq \underline{q}^{**}(\theta_2)$.*

An increase in ambiguity raises the least requirement of monitoring ability $\underline{q}^{**}$; that is, populism becomes more likely to arise. This is a contrast to the effect of an increase in risk.

Why does the result vary? The voter decides whether to reelect the incumbent based on the value of electing a new biased elite (V). Here, remember that under ambiguity, a player evaluates the payoff by using a probability measure that provides the lowest payoff among the core of a capacity. Thus, as ambiguity increases (i.e., the core of a capacity enlarges), the expected degree of bias of a new biased elite becomes higher. Then, the voter is reluctant to replace the incumbent with a new biased elite even if the incumbent's degree of bias is high. Thus, threshold β^* weakly increases with the degree of ambiguity. Since $V = -\frac{\beta^*}{1-\delta}$ holds, V weakly decreases with the degree of ambiguity. Therefore, a higher monitoring ability is necessary to prevent populism.

[34] The behavioral foundation is provided by Ghirardato and Marinacci (2002). Let $\theta_1$and θ_2 be two (not necessarily convex) capacities, and let the preference relation be $\succ_i$ ($i = 1, 2$). Then, $(\forall A \in \mathcal{F}_B)$ $\theta_2(A) \geq \theta_1(A)$ if and only if for any outcome x and act f, $x \succsim_2 f \Rightarrow x \succsim_1 f$ and $x \succ_2 f \Rightarrow x \succ_1 f$. They name this *more uncertainty averse*.

[35] Klibanoff, Marinacci, and Mukerji (2005: 1825) point out this problem: "such a separation is not evident in [...] the maxmin expected utility [..]. and the Choquet expected utility model [.]" In the smooth ambiguity model proposed by them, such separation is possible. However, in their model, people are assumed to have subjective probability over the candidates of the true distribution, and in this sense, smooth ambiguity is different from the situation where people do not have even subjective probability over the candidates of the true distribution, which is my focus. Thus, I employ the framework of Choquet/Maxmin expected utility.

7 Risk-Averse Voter

So far, the voter has been assumed to be risk-neutral. In this section, I show that the same result holds under risk-aversion so long as its degree is not high. Assume that the voter's payoff is $-|x_t - \hat{x}_t|^r$, where $r > 1$. Politicians' payoffs are defined similarly.

7.1 Equilibrium

To begin with, consider the voter's payoff when s/he elects an unbiased non-elite as the policymaker in every period. When the unbiased non-elite observes the value of $\hat{x}_t$, s/he chooses policy $\hat{x}_t$. When s/he does not observe the value of $\hat{x}_t$, s/he chooses policy x^* that minimizes $\int_{-\infty}^{\infty} |x_t - \hat{x}_t|^r dF$. Then, the voter's expected payoff when s/he elects an unbiased non-elite in every period is

$$-\frac{(1-\phi)}{1-\delta}\int_{-\infty}^{\infty} |x^* - \hat{x}_t|^r dF. \tag{10}$$

Assume the following corresponding to Assumption 4, termed Assumption 4′.

$$\max\left\{\int_0^{\bar{\beta}} \beta^r dG \middle| G \in \text{core}(\theta)\right\} > (1-\phi)\int_{-\infty}^{\infty} |x^* - \hat{x}_t|^r dF.$$

The voter's equilibrium strategy is the same as that in the basic model since it does not depend on $r = 1$. The only change from the basic model is β^{**}. A biased elite whose degree of bias is β has an incentive to choose policy mismatch β^{**} if and only if

$$\frac{\rho - (\beta - \beta^*)^r}{1-\delta} \geq \rho + \delta(1-q)\frac{\rho - (\beta - \beta^*)^r}{1-\delta} \Leftrightarrow \beta \leq \beta^{**} \equiv \beta^* + \left(\frac{q\delta\rho}{1-(1-q)\delta}\right)^{\frac{1}{r}}.$$

Given this, the correspondence to $h(\tilde{\beta})$ in the basic model is

$$\begin{aligned} h(\tilde{\beta}) = -(1-\delta(1-q))\frac{\tilde{\beta}^r}{1-\delta} - \min\Bigg\{ & -\int_0^{\tilde{\beta}} \beta^r dG - \int_{\tilde{\beta}}^{\min\left\{\tilde{\beta}+\left(\frac{q\delta\rho}{1-(1-q)\delta}\right)^{\frac{1}{r}},\bar{\beta}\right\}} \tilde{\beta}^r dG \\ & - \int_{\min\left\{\tilde{\beta}+\left(\frac{q\delta\rho}{1-(1-q)\delta}\right)^{\frac{1}{r}},\bar{\beta}\right\}}^{\bar{\beta}} \beta^r dG + \delta q\left[-\frac{1}{1-\delta}\int_0^{\tilde{\beta}} \beta^r dG - \frac{1}{1-\delta}\int_{\tilde{\beta}}^{\bar{\beta}} \tilde{\beta}^r dG\right] \Bigg| G \in \text{core}(\theta)\Bigg\}. \end{aligned}$$

Then, by using the same procedure as in Lemma 11, the existence of unique $\beta^* \in \left(0, \bar{\beta} - \left(\frac{\delta q\rho}{1-(1-q)\delta}\right)^{\frac{1}{r}}\right)$. that is the solution to $h(\tilde{\beta}) = 0$, is shown. Finally, the correspondence to Theorem 1 is obtained.

Theorem 2 *There is an equilibrium in which the voter never votes for an unbiased non-elite on the equilibrium path if and only if for β^* that satisfies $h(\tilde{\beta}) = 0$,*

$$\beta^* \leq \bar{\beta}^* \equiv (1-\phi) \int_{-\infty}^{\infty} |x^* - \hat{x}_t|^r dF \tag{11}$$

holds.

7.2 Effect of an Increase in Risk

As in Section 5, define $\bar{\beta}^*$, $\underline{q}^*$, and $\underline{q}^{**}$. Then, the result corresponding to Proposition 1 is obtained. Given this, analyze the effect of an increase in risk.

Proposition 4 *Suppose that probability distribution G_1 is a mean-preserving spread of probability distribution G_2, and that inequality (8) holds with a strong inequality when $\tilde{\beta} = \bar{\beta}^*$. In addition, assume $\rho \in (0, \bar{\rho})$, where $\bar{\rho}$ is defined in Proposition 1. Then, there is $\bar{r} > 1$ such that for any $r \in (1, \bar{r})$, $\underline{q}^{**}(G_1) \leq \underline{q}^{**}(G_2)$ holds.*[36]

An increase in risk can encourage the voter to elect a biased elite as the policymaker even when the voter hates risk. When the degree of risk-aversion is not large, the positive effect of an increase in risk, which was obtained in Section 6.1, dominates the negative effect due to risk-aversion. As a result, an increase in risk makes populism less likely to arise.

8 Concluding Remarks

In the present paper, populism was defined as a phenomenon such that voters vote for a politician, who does not have sufficient ability but is not biased, instead of a biased elite. Given this concept of populism, I constructed an infinite horizon model in which a representative voter chooses a policymaker at the beginning of each period and the elected politician implements a policy. Then, I analyzed how an increase in the uncertainty about an elite's degree of bias affects the emergence of populism. I found that an increase in risk (ambiguity) makes populism less (more) likely to arise, suggesting that an increase in ambiguity rather than in risk is a crucial source of populism.

Before closing this paper, let me see the remaining challenges for the future researches. First, I focused on stationary equilibria. How the result changes if non-stationary equilibria are taken into account is thus an important question. Second, it may be worthwhile analyzing the learning process

[36] Although I showed only the correspondence to (a), the correspondence to (b) can be obtained in a similar way

profoundly by assuming that the probability distribution is identical over time. These issues are left to later work.

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Appendices

A Assumptions on Capacity

In Section 3.2.1, the several assumptions are imposed on θ. In this section, I explain the details about the assumptions.

θ is continuous if the following two conditions hold:

$$(\forall \langle A_i \rangle_i \subseteq \mathcal{F}_B) \;\; A_1 \subseteq A_2 \subseteq A_3 \subseteq \ldots \Rightarrow \theta(\cup_i A_i) = \lim_{i \to \infty} \theta(A_i).$$

$$(\forall \langle A_i \rangle_i \subseteq \mathcal{F}_B) \;\; A_1 \supseteq A_2 \supseteq A_3 \supseteq \ldots \Rightarrow \theta(\cap_i A_i) = \lim_{i \to \infty} \theta(A_i).$$

One example, where continuity does not hold, is $\varepsilon-$contamination, whose axiomatic foundation is given by Nishimura and Ozaki (2006) and Kopylov (2009): for any $A \in \mathcal{F}_B$,

$$\theta(A) = \begin{cases} (1-\varepsilon)P_0(A) \;\; (A \neq B) \\ 1 \;\; (A = B) \end{cases},$$

where $\varepsilon \in (0, 1)$ and P_0 is a probability measure.

However, a non-continuous capacity can be approximated using a continuous capacity. To see this, consider the following approximation of $\varepsilon-$contamination, which is called $\delta-$approximation of $\varepsilon-$contamination and is provided by Nishimura and Ozaki (2004): for any $A \in \mathcal{F}_B$,

$$\theta_\delta(A) = \begin{cases} (1-\varepsilon)P_0(A) \;\; (P_0(A) \leq 1-\delta) \\ (1-\varepsilon)P_0(A) + \varepsilon[(P_0(A)-1)/\delta + 1] \;\; (P_0(A) > 1-\delta) \end{cases}.$$

(δ is different from the discount factor δ defined in Section 3.1.4.) When δ is sufficiently small, this capacity is an approximation of ε-contamination. And, this capacity satisfies continuity. In this sense, continuity is not that restrictive.

Lastly, I provide one example that satisfies the assumption that all the probability distribution functions contained in the core of θ are continuous. The example is $\delta-$approximation of $\varepsilon-$contamination discussed above. In this case, the core of θ can be written as

$$\text{core}(\theta) = \left\{ (1-\varepsilon)P_0 + \varepsilon\mu \middle| \mu \in \mathcal{M}(P_0, \delta) \right\},$$

where

$$\mathcal{M}(P_0, \delta) = \left\{ \mu \in \mathcal{M} \middle| (\forall A) \;\; \delta\mu(A) \leq P_0(A) \right\}.$$

Note that $\mathcal{M}$ is the set of all probability measures.[37] Therefore, all the probability measures contained in core(θ) assign the zero probability to any single point (i.e., continuous distribution function) so long as $\delta > 0$ and P_0 assigns the zero probability.

[37]In general, the core of a convex capacity θ is defined by $\text{core}(\theta) = \{P \in \mathcal{P} | (\forall A) \;\; P(A) \geq \theta(A)\}$, where $\mathcal{P}$ is the set of all probability charges. As mentioned in Section 3.2.1, when θ is continuous, a probability charge in core(θ) is a probability measure.

B Omitted Proofs

B.1 Proof of Lemma 1

From the argument, the expected payoff in a stage game when electing an unbiased non-elite as the policymaker in every period is $-2\int_0^\infty \hat{x}_t dF$. Thus, (2) obtains. ■

B.2 Proof of Lemma 2

(Strategy) A biased elite's strategy is the same across time from Assumption 2.

(Belief) From the capacity specified in Section 3.2.2, the beliefs about (i) a new biased elite and (ii) the incumbent biased elite, whose implemented policy has never been observed are equal to θ. This is the same as the belief about a biased elite in period 0. In addition, how the belief about the biased elite is updated after period t is the same as that in period 0 since the initial capacity is the same, and a biased elite's strategy is also the same.

Therefore, the voter's payoff must be the same. ■

B.3 Proof of Lemma 3

When $d_{\tau^*(t)} = d$ cannot be observed given the elites' equilibrium strategies, this holds from Assumption 3. Thus, consider the case where $d_{\tau^*(t)} = d$ can be observed given the elites' equilibrium strategies. Since the incumbent biased elite cannot win the election when s/he chooses policy mismatch d, only a biased elite whose degree of bias is d has an incentive to do so. Notice that such an elite exists only when $d \leq \bar{\beta}$. Hence, if $d_{\tau^*(t)=d}$ can be observed given the elites' equilibrium strategies, the voter believes that the incumbent's degree of bias is d. ■

B.4 Proof of Lemma 4

Notice that V_u is independent of s^{t-1} and t since the logic same as in Lemma 2 can be applied. Prove by contradiction. Suppose not. Then, $V_u > V$ implies that the voter votes for a new unbiased non-elite in period 0. However, this contradicts that the voter votes for a biased elite in every period on the equilibrium path. ■

B.5 Proof of Lemma 5

Step. 1: The voter (does not) reelects the incumbent biased elite if $d_{\tau^*(t)} < \beta^*$ ($d_{\tau^*(t)} > \beta^*$)

Any strategy, such that there is no $d_{\tau^*(t)} > 0$ where the voter reelects the incumbent biased elite, trivially satisfies the property. Thus, I focus on a strategy such that there is $d_{\tau^*(t)} > 0$ where the voter reelects the incumbent biased elite. Denote such $d_{\tau^*(t)}$ by d^*.

Case (i): $d^* \in (0, \bar{\beta}]$

I show that for any $d_{\tau^*(t)} \in [0, d^*]$, the voter reelects the incumbent biased elite on the equilibrium.

Since a biased elite can be reelected after implementing a policy such that $|x_t - \hat{x}_t| = d^*$, a biased elite whose bias β is d^* chooses a policy such that $|x_t - \hat{x}_t| = d^*$ on the equilibrium. Given this, from Assumption 2, d^* can be observed on the equilibrium path, and the voter expects that the incumbent biased elite will implement a policy such that $|x_t - \hat{x}_t| = d^*$ forever. Thus, the voter reelects her/him only if

$$-\frac{d^*}{1-\delta} \geq V. \tag{12}$$

Consider $d < d^*$. Suppose that there is $d < d^*$ such that the voter does not reelect the incumbent biased elite when $d_{\tau^*(t)} = d$. From Lemma 3, the voter expects that the incumbent biased elite's degree of bias β is d when $d_{\tau^*(t)} = d$. Thus, when $d_{\tau^*(t)} = d$, the voter has no incentive to deviate from the equilibrium strategy and reelect the incumbent biased elite only if $-d + \delta V \leq \max\{V, V_u\}$. From Lemma 4, this can be rewritten as $-\frac{d}{1-\delta} \leq V$. However, this contradicts inequality (12) since $d < d^*$. Thus, for any $d_{\tau^*(t)} \in [0, d^*]$, the voter reelects the incumbent biased elite on the equilibrium.

Case (ii): $d^* \in (\bar{\beta}, \infty)$ and d^* can be chosen on the equilibrium path

From Assumption 2, the voter expects that the incumbent biased elite will implement a policy such that $|x_t - \hat{x}_t| = d^*$ forever so long as the incumbent has never observed the deviation since s/he became the policymaker. Thus, the voter reelects her/him only if inequality (12) holds.

Consider $d < d^*$. Suppose that there is $d < d^*$ such that the voter does not reelect the incumbent biased elite when $d_{\tau^*(t)} = d$. The voter expects that the incumbent biased elite's degree of bias β is $\min\{d, \bar{\beta}\}$ when $d_{\tau^*(t)} = d$. Then, using the same procedure as in (i), I can show that for any $d_{\tau^*(t)} \in [0, d^*]$, the voter reelects the incumbent biased elite on the equilibrium.

Case (iii): $d^* \in (\bar{\beta}, \infty)$ and d^* cannot be chosen on the equilibrium path

This implies that any biased elite has no incentive to choose a policy such that $|x_t - \hat{x}_t| = d^*$. Thus, the same outcome can be sustained by the voting strategy such that the voter does not reelect the incumbent biased elite when $d_{\tau^*(t)} = d^*$. Thus, it is unnecessary to take into account case (iii).

From (i) to (iii), every outcome sustained by equilibria satisfying Assumptions 1-4 (if exists) can be constructed by the voting strategy such that he voter (does not) reelects the incumbent biased elite if $d_{\tau^*(t)} < \beta^* (d_{\tau^*(t)} > \beta^*)$.

Step. 2: $\beta^* < \bar{\beta}$ holds

Prove by contradiction. When $\beta^* \geq \bar{\beta}$, all biased elites whose $\beta \in [0, \bar{\beta})$ choose their own ideal policies if elected. Thus, the voter's payoff when s/he follows this voting strategy is (3). From Assumption 4, (3) < (2), and so in this case, such voting strategy does not constitute an equilibrium. Therefore, $\beta^* < \bar{\beta}$ holds.

Step. 3: The voter reelects the incumbent biased elite if $d_{\tau^*(t)} = \beta^*$

Prove by contradiction. Suppose that the voter does not reelect the incumbent biased elite if $d_{\tau^*(t)} = \beta^*$. Since $\rho, \delta, q > 0$ hold, there are biased elites whose $\beta > \beta^*$ and who choose a policy mismatch which is smaller than β^*. However, there is no optimal policy these biased elites should choose because for any policy mismatch which is smaller than β^*, there is a policy mismatch which is closer to β^* and is better for them. Thus, there is no such equilibrium. Therefore, in the equilibrium, the voter reelects the incumbent biased elite if $d_{\tau^*(t)} = \beta^*$.

Therefore, I obtain the lemma. ■

B.6 Proof of Lemma 7

(i) $-\frac{\beta^*}{1-\delta} < V$ does not hold.

The voter has no incentive to deviate from the strategy on the equilibrium path when $d_{\tau^*(t)} = \beta^*$ only if $-\frac{\beta^*}{1-\delta} \geq V$. Thus, $-\frac{\beta^*}{1-\delta} < V$ does not hold.

(ii) $-\frac{\beta^*}{1-\delta} > V$ does not hold.

When $-\frac{\beta^*}{1-\delta} > V$ holds, there is $\beta' \in (\beta^*, \beta^{**})$ such that $-\frac{\beta'}{1-\delta} > V$ does not hold. Then, from Lemma 3, the voter expects that the incumbent biased elite's bias is β' when $d_{\tau^*(t)} = \beta'$ and s/he has never observed deviation from equilibrium. Thus, by one-shot deviation, the voter obtains the utility: $-\beta' + \delta V$. This must be smaller than or equal to $\max\{V_u, V\}$ i.e., $-\frac{\beta^*}{1-\delta} \leq V$ must hold from Lemma 4. This is contradiction.

From (i) and (ii), $-\frac{\beta^*}{1-\delta} = V$. ■

B.7 Proof of Lemma 8

(i) "Only if" part: Suppose that there is an equilibrium in which the voter never votes for an unbiased non-elite on the equilibrium path when the iterated payoff is employed. Then, the same equilibrium outcome can be created using the strategy specified in Section 4.3.2 and the belief system specified in Section 4.3.3. Consider the equilibrium with these strategies and beliefs. Since rectangularity holds, the iterated payoff is equivalent to the non-iterated payoff. Thus, the equilibrium is sustained also when the non-iterated payoff is employed.

(ii) "If" part: Suppose that there is an equilibrium in which the voter never votes for an unbiased non-elite on the equilibrium path when the non-iterated payoff is employed. Observe that the proofs of Lemmas 2-7 do not depend on the fact that the payoff is iterated one. Thus, the same equilibrium outcome can be created using the strategy specified in Section 4.3.2 and the belief system specified in Section 4.3.3. Consider the equilibrium with these strategies and beliefs. Since rectangularity holds, the iterated payoff is equivalent to the non-iterated payoff. Thus, the equilibrium is sustained also when the iterated payoff is employed.

In summary, Lemma 8 is proven. ■

B.8 Proof of Lemma 9

(i) $h(\tilde{\beta})$ is a decreasing function for $\tilde{\beta} \in \left[0, \bar{\beta} - \frac{\delta q \rho}{1-(1-q)\delta}\right]$

Denote

$$J(\tilde{\beta}, G|q) \equiv \int_0^{\tilde{\beta}} \beta dG + \int_{\tilde{\beta}}^{\min\left\{\tilde{\beta}+\frac{q\delta\rho}{1-(1-q)\delta}, \bar{\beta}\right\}} \tilde{\beta} dG + \int_{\min\left\{\tilde{\beta}+\frac{q\delta\rho}{1-(1-q)\delta}, \bar{\beta}\right\}}^{\bar{\beta}} \beta dG + \delta q \left[\frac{1}{1-\delta}\int_0^{\tilde{\beta}} \beta dG + \frac{1}{1-\delta}\int_{\tilde{\beta}}^{\bar{\beta}} \tilde{\beta} dG\right]. \tag{13}$$

Using this, let

$$G_{\tilde{\beta}+\varepsilon} \in \arg\min\left\{-J(\tilde{\beta}+\varepsilon|q)\middle| G \in \text{core}(\theta)\right\}.$$

Then, for any $\varepsilon \in \left(0, \bar{\beta} - \frac{\delta q \rho}{1-(1-q)\delta}\right)$,

$$
\begin{aligned}
&h(\tilde{\beta}+\varepsilon) - h(\tilde{\beta}) \\
&\quad < -(1-\delta(1-q))\frac{\varepsilon}{1-\delta} + \int_0^{\tilde{\beta}+\varepsilon} \beta dG_{\tilde{\beta}+\varepsilon} + \int_{\tilde{\beta}+\varepsilon}^{\tilde{\beta}+\frac{\delta q\rho}{1-(1-q)\delta}} (\tilde{\beta}+\varepsilon) dG_{\tilde{\beta}+\varepsilon} + \int_{\tilde{\beta}+\varepsilon+\frac{\delta q\rho}{1-(1-q)\delta}}^{\bar{\beta}} \beta dG_{\tilde{\beta}+\varepsilon} \\
&\qquad + \frac{\delta q}{1-\delta}\int_0^{\tilde{\beta}+\varepsilon} \beta dG_{\tilde{\beta}+\varepsilon} + \frac{\delta q}{1-\delta}\int_{\tilde{\beta}+\varepsilon}^{\bar{\beta}} (\tilde{\beta}+\varepsilon) dG_{\tilde{\beta}+\varepsilon} - \int_0^{\tilde{\beta}} \beta dG_{\tilde{\beta}+\varepsilon} - \int_{\tilde{\beta}}^{\tilde{\beta}+\frac{\delta q\rho}{1-(1-q)\delta}} \tilde{\beta} dG_{\tilde{\beta}+\varepsilon} - \int_{\tilde{\beta}+\frac{\delta q\rho}{1-(1-q)\delta}}^{\bar{\beta}} \beta dG_{\tilde{\beta}+\varepsilon} \\
&\qquad - \frac{\delta q}{1-\delta}\int_0^{\tilde{\beta}} \beta dG_{\tilde{\beta}+\varepsilon} - \frac{\delta q}{1-\delta}\int_{\tilde{\beta}}^{\bar{\beta}} \tilde{\beta} dG_{\tilde{\beta}+\varepsilon} \\
&\quad < -(1-\delta(1-q))\frac{\varepsilon}{1-\delta} + (\tilde{\beta}+\varepsilon)\left[G_{\tilde{\beta}+\varepsilon}(\tilde{\beta}+\varepsilon) - G_{\tilde{\beta}+\varepsilon}(\tilde{\beta})\right] \\
&\qquad + \tilde{\beta}\left[G_{\tilde{\beta}+\varepsilon}\left(\tilde{\beta}+\varepsilon+\frac{\delta q\rho}{1-(1-q)\delta}\right) - G_{\tilde{\beta}+\varepsilon}\left(\tilde{\beta}+\frac{\delta q\rho}{1-(1-q)\delta}\right)\right] - \tilde{\beta}\left[G_{\tilde{\beta}+\varepsilon}(\tilde{\beta}+\varepsilon) - G_{\tilde{\beta}+\varepsilon}(\tilde{\beta})\right] \\
&\qquad + \varepsilon\left[G_{\tilde{\beta}+\varepsilon}\left(\tilde{\beta}+\varepsilon+\frac{\delta q\rho}{1-(1-q)\delta}\right) - G_{\tilde{\beta}+\varepsilon}\left(\tilde{\beta}+\varepsilon\right)\right] \\
&\qquad - \left(\tilde{\beta}+\frac{\delta q\rho}{1-(1-q)\delta}\right)\left[G_{\tilde{\beta}+\varepsilon}\left(\tilde{\beta}+\varepsilon+\frac{\delta q\rho}{1-(1-q)\delta}\right) - G_{\tilde{\beta}+\varepsilon}\left(\tilde{\beta}+\frac{\delta q\rho}{1-(1-q)\delta}\right)\right] \\
&\qquad + \frac{\delta q}{1-\delta}(\tilde{\beta}+\varepsilon)\left[G_{\tilde{\beta}+\varepsilon}(\tilde{\beta}+\varepsilon) - G_{\tilde{\beta}+\varepsilon}(\tilde{\beta})\right] - \frac{\delta q}{1-\delta}\tilde{\beta}\left[G_{\tilde{\beta}+\varepsilon}(\tilde{\beta}+\varepsilon) - G_{\tilde{\beta}+\varepsilon}(\tilde{\beta})\right] + \frac{\delta q}{1-\delta}\varepsilon\left[1 - G_{\tilde{\beta}+\varepsilon}(\tilde{\beta}+\varepsilon)\right] \\
&\quad = \varepsilon\left\{\left[G_{\tilde{\beta}+\varepsilon}\left(\tilde{\beta}+\varepsilon+\frac{\delta q\rho}{1-(1-q)\delta}\right) - G_{\tilde{\beta}+\varepsilon}(\tilde{\beta})\right] + \frac{\delta q}{1-\delta}\left[1 - G_{\tilde{\beta}+\varepsilon}(\tilde{\beta})\right] - \frac{(1-\delta(1-q))}{1-\delta}\right\} \\
&\qquad - \frac{\delta q\rho}{1-(1-q)\delta}\left[G_{\tilde{\beta}+\varepsilon}\left(\tilde{\beta}+\varepsilon+\frac{\delta q\rho}{1-(1-q)\delta}\right) - G_{\tilde{\beta}+\varepsilon}\left(\tilde{\beta}+\frac{\delta q\rho}{1-(1-q)\delta}\right)\right]. \qquad (14)
\end{aligned}
$$

The first inequality comes from the nature of $\min\{\cdot\}$. Here, the first term of (14) is negative since

$$G_{\tilde{\beta}+\varepsilon}\left(\tilde{\beta}+\varepsilon+\frac{\delta q\rho}{1-(1-q)\delta}\right) - G_{\tilde{\beta}+\varepsilon}(\tilde{\beta}) + \frac{\delta q}{1-\delta}\left[1 - G_{\tilde{\beta}+\varepsilon}(\tilde{\beta})\right] - \frac{(1-\delta(1-q))}{1-\delta} < 1 + \frac{\delta q}{1-\delta} - \frac{(1-\delta(1-q))}{1-\delta} = 0.$$

Also, the second term is obviously negative. Thus, (14)<0, and so $h(\tilde{\beta}+\varepsilon) - h(\tilde{\beta}) < 0$. ■

(ii) $h(\tilde{\beta}) < 0$ for any $\tilde{\beta} \in \left[\bar{\beta} - \frac{\delta q\rho}{1-(1-q)\delta}, \bar{\beta}\right]$

For $\tilde{\beta} \in \left[\bar{\beta} - \frac{\delta q\rho}{1-(1-q)\delta}, \bar{\beta}\right]$,

$$
\begin{aligned}
h(\tilde{\beta}) &= -(1-\delta(1-q))\frac{\tilde{\beta}}{1-\delta} - \min\left\{-\frac{1-\delta(1-q)}{1-\delta}\left[\int_0^{\tilde{\beta}} \beta dG + \int_{\tilde{\beta}}^{\bar{\beta}} \tilde{\beta} dG\right] \middle| G \in \mathrm{core}(\theta)\right\} \\
&= -\frac{(1-\delta(1-q))}{1-\delta}\left\{-\tilde{\beta} + \max\left\{\int_0^{\tilde{\beta}} \beta dG + \int_{\tilde{\beta}}^{\bar{\beta}} \tilde{\beta} dG \middle| G \in \mathrm{core}(\theta)\right\}\right\} < 0.
\end{aligned}
$$

The last inequality holds since

$$\tilde{\beta} < \max\left\{ \int_0^{\tilde{\beta}} \beta dG + \int_{\tilde{\beta}}^{\bar{\beta}} \tilde{\beta} dG \middle| G \in \mathrm{core}(\theta) \right\}$$

holds because of the fact that G is full support. ■

B.9 Proof of Lemma 10

Pick up a $\tilde{\beta}$ and denote it by b. Then, what I should show is that the following holds:

$$(\forall \varepsilon > 0)\ \ (\exists \gamma > 0)\ \ (\forall \tilde{\beta} \in (0, \bar{\beta}))\ \ [|\tilde{\beta} - b| < \gamma \Rightarrow |I(\tilde{\beta}) - I(b)| < \varepsilon]. \tag{15}$$

Fix $\varepsilon > 0$.

(i) When $\tilde{\beta} > b$:

First, consider this case. Define $I'_a(\tilde{\beta})$ as $J(\tilde{\beta}, G_a)$ where $G_a \in \arg\min_{G \in \mathrm{core}(\theta)} J(a, G)$. Then, since $I'_a(\tilde{\beta})$ is obviously a continuous function,

$$(\exists \gamma > 0)\ \ (\forall \tilde{\beta} \in (0, \bar{\beta}))\ \ [|\tilde{\beta} - b| < \gamma \Rightarrow |I'(\tilde{\beta}) - I'(b)| < \varepsilon]$$

holds. Denote this γ by $\bar{\gamma}$. Here,

$$|I'(\tilde{\beta}) - I'(b)| = |I'(\tilde{\beta}) - I(b)| \geq |I(\tilde{\beta}) - I(b)|.$$

Thus,

$$(\exists \bar{\gamma} > 0)\ \ (\forall \tilde{\beta} \in (b, \bar{\beta}))\ \ [|\tilde{\beta} - b| < \gamma \Rightarrow |I(\tilde{\beta}) - I(b)| < \varepsilon].$$

(ii) When $\tilde{\beta} < b$:

Next, consider the case where $\tilde{\beta}$ is smaller than b. Since I'_a is a continuous function, there is $a < b$ such that $I'_a(a) - I'_a(b) < \varepsilon$. Here, for any $\tilde{\beta} \in (b - \underline{\gamma}, b)$ where $\underline{\gamma} = b - a$,

$$|I'_a(a) - I'_a(b)| = |I(a) - I'_a(b)| \geq |I(a) - I(b)| \geq |I(\tilde{\beta}) - I(b)|$$

holds. Thus,

$$(\exists \underline{\gamma} > 0)\ \ (\forall \tilde{\beta} \in (0, b))\ \ [|\tilde{\beta} - b| < \gamma \Rightarrow |I(\tilde{\beta}) - I(b)| < \varepsilon].$$

Combining (i) and (ii),

$$(\exists \min\{\underline{\gamma}, \bar{\gamma}\} > 0)\ \ (\forall \tilde{\beta} \in (b, \bar{\beta}))\ \ [|\tilde{\beta} - b| < \gamma \Rightarrow |I(\tilde{\beta}) - I(b)| < \varepsilon].$$

Thus, (15) holds. ■

B.10 Proof of Lemma 11

To begin with, from Lemma 9, $h(\tilde{\beta}) < 0$ holds for any $\tilde{\beta} \in \left[\bar{\beta} - \frac{\delta q \rho}{1-(1-q)\delta}, \bar{\beta}\right]$. Thus, $\beta^* < \bar{\beta} - \frac{\delta q \rho}{1-(1-q)\delta}$. Therefore, I focus on $\tilde{\beta} \in \left[0, \bar{\beta} - \frac{\delta q \rho}{1-(1-q)\delta}\right)$.

Here, $h(\tilde{\beta})$ is decreasing with $\tilde{\beta}$ for any $\tilde{\beta} \in \left[0, \bar{\beta} - \frac{\delta q \rho}{1-(1-q)\delta}\right)$ from Lemma 9. And,

$$h(0) = \max\left\{ \int_{\frac{\delta q \rho}{1-(1-q)\delta}}^{\bar{\beta}} \beta dG \middle| G \in \mathrm{core}(\theta) \right\} > 0.$$

Thus, from the continuity of h (Lemma 10), there is a unique $\beta^* \in \left(0, \bar{\beta} - \frac{\delta q \rho}{1-(1-q)\delta}\right)$ which satisfies equation (5) if and only if $h\left(\bar{\beta} - \frac{\delta q \rho}{1-(1-q)\delta}\right) < 0$. Actually this holds from Lemma 9. Therefore, there is a unique $\beta^* \in \left(0, \bar{\beta} - \frac{\delta q \rho}{1-(1-q)\delta}\right)$ which satisfies equation (5). ■

B.11 Proof of Theorem 1

Suppose that there is an equilibrium in which the voter never votes for an unbiased non-elite on the equilibrium path. From the previous discussions, the voter has no incentive to elect a biased elite who is different from the elite the voter must vote for in the equilibrium. Thus, it suffices to examine the voter's one-shot deviation such that the voter votes for a new unbiased non-elite.

Case (i): the incumbent is a biased elite and $d_{\tau^*(t)} \in [0, \beta^*]$.

The voter has no incentive to vote for a new unbiased non-elite if and only if

$$-\frac{d_{\tau^*(t)}}{1-\delta} \geq -2(1-\phi) \int_0^\infty \hat{x}_t dF + \delta V.$$

This holds for any $d_{\tau^*(t)} \in [0, \beta^*]$ if and only if (7) holds.

Case (ii): the incumbent is biased elite and $d_{\tau^*(t)} = \emptyset$.

The voter has no incentive to vote for a new unbiased non-elite if and only if

$$V \geq -2(1-\phi) \int_0^\infty \hat{x}_t dF + \delta V.$$

This is written as condition (7) because $V = -\frac{\beta^*}{1-\delta}$ holds.

Case (iii): the voter must vote for a new biased elite based on her/his equilibrium strategy.

The voter has no incentive to vote for a new unbiased non-elite if and only if

$$V \geq -2(1-\phi) \int_0^\infty \hat{x}_t dF + \delta V.$$

This is written as condition (7).

From cases 1-3, if and only if condition (7) holds, such an equilibrium exists. ■

B.12 Proof of Lemma 12

Suppose that $q_1 > q_2$. The objective is to show that $\beta^*(q_1) < \beta^*(q_2)$ holds.

From Lemma 9, $h(\tilde{\beta}) < 0$ holds for any $\tilde{\beta} > \beta^*$. This implies that when $h(\tilde{\beta}|q^1) < 0$ is satisfied for any $\tilde{\beta} \geq \beta^*(q^2)$, $\beta^*(q^1) < \beta^*(q^2)$ holds. Therefore, my task is to show that $h(\tilde{\beta}|q^1) < 0$ is satisfied for any $\tilde{\beta} \geq \beta^*(q^2)$.

When $h(\tilde{\beta}|q_1) < h(\tilde{\beta}|q_2)$, this trivially holds since $h(\tilde{\beta}|q_2) \leq 0$. Let $G_{q_1} \in \arg\min \left\{ -J(\tilde{\beta}|q_1) \middle| G \in \text{core}(\theta) \right\}$, where $J(\tilde{\beta}|q)$ is defined by (13). Indeed, $h(\tilde{\beta}|q_1) < h(\tilde{\beta}|q_2)$ holds since

$$\begin{aligned}
h(\tilde{\beta}|q_1) - h(\tilde{\beta}|q_2) \leq & -\delta(q_1 - q_2)\frac{\tilde{\beta}}{1-\delta} + \int_0^{\tilde{\beta}} \beta dG_{q_1} + \int_{\tilde{\beta}}^{\min\left\{\tilde{\beta}+\frac{q\delta\rho}{1-(1-q_1)\delta}, \bar{\beta}\right\}} \tilde{\beta} dG_{q_1} + \int_{\min\left\{\tilde{\beta}+\frac{q_1\delta\rho}{1-(1-q_1)\delta}, \bar{\beta}\right\}}^{\bar{\beta}} \beta dG_{q_1} \\
& + \delta q_1 \left[\frac{1}{1-\delta} \int_0^{\tilde{\beta}} \beta dG_{q_1} + \frac{1}{1-\delta} \int_{\tilde{\beta}}^{\bar{\beta}} \tilde{\beta} dG_{q_1} \right] \\
& - \int_0^{\tilde{\beta}} \beta dG_{q_1} - \int_{\tilde{\beta}}^{\min\left\{\tilde{\beta}+\frac{q_2\delta\rho}{1-(1-q_2)\delta}, \bar{\beta}\right\}} \tilde{\beta} dG_{q_1} - \int_{\min\left\{\tilde{\beta}+\frac{q_2\delta\rho}{1-(1-q_2)\delta}, \bar{\beta}\right\}}^{\bar{\beta}} \beta dG_{q_1} \\
& - \delta q_2 \left[\frac{1}{1-\delta} \int_0^{\tilde{\beta}} \beta dG_{q_1} + \frac{1}{1-\delta} \int_{\tilde{\beta}}^{\bar{\beta}} \tilde{\beta} dG_{q_1} \right] \\
= & -\delta(q_1 - q_2)\frac{\tilde{\beta}}{1-\delta} + \int_{\min\left\{\tilde{\beta}+\frac{q_2\delta\rho}{1-(1-q_2)\delta}, \bar{\beta}\right\}}^{\min\left\{\tilde{\beta}+\frac{q_1\delta\rho}{1-(1-q_1)\delta}, \bar{\beta}\right\}} (\tilde{\beta} - \beta) dG_{q_1} + \frac{\delta(q_1 - q_2)}{1-\delta} \left[\int_0^{\tilde{\beta}} \beta dG_{q_1} + \int_{\tilde{\beta}}^{\bar{\beta}} \tilde{\beta} dG_{q_1} \right] \\
= & -\frac{\delta(q_1 - q_2)}{1-\delta} \left\{ \tilde{\beta} - \left[\int_0^{\tilde{\beta}} \beta dG_{q_1} + \int_{\tilde{\beta}}^{\bar{\beta}} \tilde{\beta} dG_{q_1} \right] \right\} + \int_{\min\left\{\tilde{\beta}+\frac{q_2\delta\rho}{1-(1-q_2)\delta}, \bar{\beta}\right\}}^{\min\left\{\tilde{\beta}+\frac{q_1\delta\rho}{1-(1-q_1)\delta}, \bar{\beta}\right\}} (\tilde{\beta} - \beta) dG_{q_1} \qquad (16)
\end{aligned}$$

Here, in the second equality, I use the fact that

$$\tilde{\beta} + \frac{q_2\delta\rho}{1-(1-q_2)\delta} < \tilde{\beta} + \frac{q_1\delta\rho}{1-(1-q_1)\delta}.$$

Then, the first term of (16) is negative since G_{q_1} has full-support and the second term of (16) is obviously non-positive. In summary, $h(\tilde{\beta}|q_1) - h(\tilde{\beta}|q_2) < 0$.

Therefore, $\beta^*(q_1) < \beta^*(q_2)$ holds. ■

B.13 Proof of Proposition 1

I show only that there is unique $\underline{q}^{**}$ because the other part trivially holds given Lemma 12. To prove this, it suffices to show the existence of unique $\underline{q}^*$.

$h(\bar{\beta}^*) > 0$ when $q = 0$ since

$$h(\bar{\beta}^*|q=0) = -(1-\delta)\frac{\bar{\beta}^*}{1-\delta} - \min\left\{ -\int_0^{\bar{\beta}} \beta dG \middle| G \in \text{core}(\theta) \right\} = -\bar{\beta}^* + \max\left\{ \int_0^{\bar{\beta}} \beta dG \middle| G \in \text{core}(\theta) \right\} > 0.$$

The last inequality comes from Assumption 4. In addition, $h(\tilde{\beta})$ is decreasing with $q \geq 0$, and obviously $h(\tilde{\beta})$ is continuous with respect to q. Thus, there is unique $\underline{q}^* \geq 0$. ■

B.14 Proof of Lemma 14

$$h(\tilde{\beta}) = -\frac{1-\delta(1-q)}{1-\delta}\tilde{\beta} + \int_0^{\bar{\beta}} \beta dG - \int_{\tilde{\beta}}^{\min\left\{\tilde{\beta}-\frac{q\delta\rho}{1-(1-q)\delta}, \bar{\beta}\right\}} (\beta - \tilde{\beta}) dG + \frac{\delta q}{1-\delta} \left[\int_0^{\tilde{\beta}} \beta dG + \int_{\tilde{\beta}}^{\bar{\beta}} \tilde{\beta} dG \right]. \qquad (17)$$

Here,

$$\int_0^{\tilde{\beta}} \beta dG = \beta G(\beta)|_0^{\tilde{\beta}} - \int_0^{\tilde{\beta}} G(\beta)d\beta = \tilde{\beta}G(\tilde{\beta}) - \int_0^{\tilde{\beta}} G(\beta)d\beta. \tag{18}$$

since G is differentiable. Thus, by substituting (18) into (17),

$$(17) = -\tilde{\beta} + \int_0^{\tilde{\beta}} \beta dG + \int_{\tilde{\beta}}^{\min\left\{\tilde{\beta} - \frac{q\delta\rho}{1-(1-q)\delta}, \bar{\beta}\right\}} (\beta - \tilde{\beta})dG - \frac{\delta q}{1-\delta}\int_0^{\tilde{\beta}} G(\beta)d\beta. \blacksquare$$

B.15 Proof of Proposition 2

B.15.1 Proof of Proposition 2 (a)

To begin with, I show that if $f(\bar{\beta}^*|q, G_1) < f(\bar{\beta}^*|qG_2)$ holds for any $q \in [\underline{q}, 1]$, $\underline{q}^{**}(G_1) \le \underline{q}^{**}(G_2)$.

Case (i): $\underline{q}^{**}(G_2) = \bar{q}$.

In this case, $\underline{q}^{**}(G_1) \le \underline{q}^{**}(G_2)$ always holds by definition.

Case (ii): $\underline{q}^{**}(G_2) \in (\underline{q}, \bar{q})$.

For any $q \in [0, \underline{q}^*(G_1)]$, $h(\bar{\beta}^*|q, G_1) \ge 0$ since $h(\tilde{\beta})$ is decreasing with q. Thus, when $h(\bar{\beta}^*|\underline{q}^*(G_2), G_1) < 0$, $\underline{q}^{**}(G_1) < \underline{q}^{**}(G_2)$ holds. Here, $h(\bar{\beta}^*|\underline{q}^*(G_2), G_1) < 0$ is equivalent to $h(\bar{\beta}^*|\underline{q}^*(G_2), G_1) < h(\bar{\beta}^*|\underline{q}^*(G_2), G_2)$ since $h(\bar{\beta}^*|\underline{q}^*(G_2), G_2) = 0$. Therefore, when $h(\bar{\beta}^*|q, G_1) < h(\bar{\beta}^*|qG_2)$ holds for any $q \in [\underline{q}, 1]$, $h(\bar{\beta}^*|\underline{q}^*(G_2), G_1) < 0$ is satisfied.

Case (iii): $\underline{q}^{**}(G_2) = \underline{q}$.

$\underline{q}^{**} = \underline{q}$ holds if and only if $h(\tilde{\beta}|q, G) < 0$ holds for any $q \in (\underline{q}, 1]$. Thus, $h(\tilde{\beta}|q, G_2) < 0$ holds for any $q \in (\underline{q}, 1]$. Therefore, when $h(\tilde{\beta}|q, G_1) < h(\bar{\beta}^*|q, G_2)(< 0)$ is satisfied for any $q \in (\underline{q}, 1]$, $\underline{q}^{**}(G_1) = \underline{q} = \underline{q}^{**}(G_2)$ is obtained.

From (i) to (iii), if $h(\bar{\beta}^*|q, G_1) < h(\bar{\beta}^*|q, G_2)$ holds for any $q \in [\underline{q}, 1]$, $\underline{q}^{**}(G_1) \le \underline{q}^{**}(G_2)$. Threfore, it suffices to show that $h(\bar{\beta}^*|q, G_1) < h(\bar{\beta}^*|q, G_2)$ holds for any $q \in [\underline{q}, 1]$.

Using the expression of $h(\tilde{\beta})$ derived in Lemma 14,

$$\begin{aligned} h(\bar{\beta}^*|q, G_1) - h(\bar{\beta}^*|q, G_2) = &-\frac{\delta q}{1-\delta}\left[\int_0^{\tilde{\beta}} G_1(\beta)d\beta - \int_0^{\tilde{\beta}} G_2(\beta)d\beta\right] \\ &+ \left[-\int_{\tilde{\beta}}^{\min\left\{\tilde{\beta}+\frac{q\delta\rho}{1-(1-q)\delta}, \bar{\beta}\right\}} (\beta - \bar{\beta})dG_1 + \int_{\tilde{\beta}}^{\min\left\{\tilde{\beta}+\frac{q\delta\rho}{1-(1-q)\delta}, \bar{\beta}\right\}} (\beta - \bar{\beta})dG_2\right]. \end{aligned} \tag{19}$$

Note that $\int_0^{\bar{\beta}} \beta dG_1 = \int_0^{\bar{\beta}} \beta dG_2$ from the definition of mean-preserving spread. I use this fact in the above.

Here,

$$(19) \le -\frac{\delta q}{1-\delta}\left[\int_0^{\tilde{\beta}} G_1(\beta)d\beta - \int_0^{\tilde{\beta}} G_2(\beta)d\beta\right] + \int_{\tilde{\beta}}^{\min\left\{\tilde{\beta}+\frac{q\delta\rho}{1-(1-q)\delta}, \bar{\beta}\right\}} (\beta - \bar{\beta})|g_1(\beta) - g_2(\beta)|d\beta. \tag{20}$$

The first term of (20) is independent of ρ and negative. On the other hand, the second term of (20) is weakly decreasing with ρ, and it goes to zero as ρ goes to zero since

$$\tilde{\beta} + \frac{q\delta\rho}{1-(1-q)\delta} \to \tilde{\beta}.$$

Thus, for each $q \in [\underline{q}, 1]$, there is $\bar{\rho}(q) > 0$ such that for $\rho \in (0, \bar{\rho}(q)]$, $h(\bar{\beta}^*|q, G_1) < h(\bar{\beta}^*|qG_2)$ holds. Take the minimum of $\bar{\rho}(q)$, and denote it by $\bar{\rho}$. Then, for $\rho \in (0, \bar{\rho}]$, $h(\bar{\beta}^*|q, G_1) < h(\bar{\beta}^*|qG_2)$ holds for any $q \in [\underline{q}, 1]$. Therefore, for $\rho \in (0, \bar{\rho}]$, $\underline{q}^{**}(G_1) \leq \underline{q}^{**}(G_2)$. ■

B.15.2 Proof of Proposition 2 (b)

As in the proof of Proposition 2 (a), if $h(\bar{\beta}^*|\underline{q}^*(G_1), G_1) \geq h(\bar{\beta}^*|\underline{q}^*(G_1), G_2)$ holds, $\underline{q}^*(G_1) \geq \underline{q}^*(G_2)$. Thus, it suffices to prove that $h(\bar{\beta}^*|\underline{q}^*(G_1), G_1) \geq h(\bar{\beta}^*|\underline{q}^*(G_1), G_2)$ holds. Here, from equation (19), if

$$\int_{\bar{\beta}^*}^{\min\left\{\bar{\beta}^*+\frac{\underline{q}^*(G_1)\delta\rho}{1-(1-\underline{q}^*(G_1))\delta}, \bar{\beta}\right\}} (\beta - \bar{\beta}^*)dG_2 \geq \int_{\bar{\beta}^*}^{\min\left\{\bar{\beta}^*+\frac{\underline{q}^*(G_1)\delta\rho}{1-(1-\underline{q}^*(G_1))\delta}, \bar{\beta}\right\}} (\beta - \bar{\beta}^*)dG_1 \tag{21}$$

holds, $h(\bar{\beta}^*|\underline{q}^*(G_1), G_1) \geq h(\bar{\beta}^*|\underline{q}^*(G_1), G_2)$ automatically holds.

(i) Case where condition (i) is satisfied:

In this case, inequality (21) is equivalent to

$$\int_{\bar{\beta}^*}^{\bar{\beta}^*+\frac{\underline{q}^*(G_1)\delta\rho}{1-(1-\underline{q}^*(G_1))\delta}} (\beta - \bar{\beta}^*)dG_2 \geq \int_{\bar{\beta}^*}^{\bar{\beta}^*+\frac{\underline{q}^*(G_1)\delta\rho}{1-(1-\underline{q}^*(G_1))\delta}} (\beta - \bar{\beta}^*)dG_1. \tag{22}$$

Here, from condition (i), $g_2(\beta) \geq g_1(\beta)$ holds for any $\beta \in \left[\bar{\beta}^*, \bar{\beta}^* + \frac{\underline{q}^*(G_1)\delta\rho}{1-(1-\underline{q}^*(G_1))\delta}\right]$. Therefore, inequality (22) holds since $\beta - \beta^* \geq 0$.

(ii) Case where condition (ii) is satisfied

In this case, inequality (21) is also equivalent to inequality (22). Here,

$$\int_{\bar{\beta}^*}^{\bar{\beta}^*+\frac{\underline{q}^*(G_1)\delta\rho}{1-(1-\underline{q}^*(G_1))\delta}} (\beta - \bar{\beta}^*)dG = \int_0^{\bar{\beta}} (\beta - \bar{\beta}^*)dG - \int_0^{\bar{\beta}^*} (\beta - \bar{\beta}^*)dG - \int_{\bar{\beta}^*+\frac{\underline{q}^*(G_1)\delta\rho}{1-(1-\underline{q}^*(G_1))\delta}}^{\bar{\beta}} (\beta - \bar{\beta}^*)dG.$$

Given the fact that $\int_0^{\bar{\beta}}(\beta - \bar{\beta}^*)dG_1 = \int_0^{\bar{\beta}}(\beta - \bar{\beta}^*)dG_2$ from the definition of the mean-preserving spread, inequality (22) holds if and only if

$$\int_0^{\bar{\beta}^*} (\beta - \bar{\beta}^*)dG_1 + \int_{\bar{\beta}^*+\frac{\underline{q}^*(G_1)\delta\rho}{1-(1-\underline{q}^*(G_1))\delta}}^{\bar{\beta}} (\beta - \bar{\beta}^*)dG_1 \geq \int_0^{\bar{\beta}^*} (\beta - \bar{\beta}^*)dG_2 + \int_{\bar{\beta}^*+\frac{\underline{q}^*(G_1)\delta\rho}{1-(1-\underline{q}^*(G_1))\delta}}^{\bar{\beta}} (\beta - \bar{\beta}^*)dG_2. \tag{23}$$

Here, from condition (ii), $g_2(\beta) \leq g_1(\beta)$ and $\beta - \bar{\beta}^* > 0$ hold for any $\beta \in \left[\bar{\beta}^* + \frac{\underline{q}^*(G_1)\delta\rho}{1-(1-\underline{q}^*(G_1))\delta}, \bar{\beta}\right]$.

Thus, inequality (23) holds if

$$\int_0^{\bar{\beta}^*}(\beta-\bar{\beta}^*)dG_1+\int_{\bar{\beta}-\bar{\beta}^*}^{\bar{\beta}}(\beta-\bar{\beta}^*)dG_1\geq\int_0^{\bar{\beta}^*}(\beta-\bar{\beta}^*)dG_2+\int_{\bar{\beta}-\bar{\beta}^*}^{\bar{\beta}}(\beta-\bar{\beta}^*)dG_2. \tag{24}$$

Here, due to the symmetry of G, $g(\beta)=g(\bar{\beta}-\beta)$ holds for any $\beta\in[0,\bar{\beta}^*]$. Using this, the left-hand side minus the right-hand side of inequality (24) is equivalent to

$$\int_0^{\bar{\beta}^*}(\beta-\bar{\beta}^*)(g_1(\beta)-g_2(\beta))d\beta+\int_0^{\bar{\beta}^*}(\bar{\beta}-\beta-\bar{\beta}^*)(g_1(\beta)-g_2(\beta))d\beta.$$

Now, this is non-negative since $|\beta-\bar{\beta}^*|<|\bar{\beta}-\beta-\bar{\beta}^*|$ for any $\beta\in[0,\bar{\beta}^*]$ from condition (ii). Therefore, inequality (24) holds. As a result, I have inequality (23). ■

B.16 Proof of Proposition 3

As in the proof of Proposition 2, if $h(\bar{\beta}^*|\underline{q}^*(\theta_2),\theta_1)\geq h(\bar{\beta}^*|\underline{q}^*(\theta_2),\theta_2)$ holds, $\underline{q}^*(\theta_1)\geq\underline{q}^*(\theta_2)$. Thus, it suffices to prove that $h(\bar{\beta}^*|\underline{q}^*(\theta_2),\theta_1)\geq h(\bar{\beta}^*|\underline{q}^*(\theta_2),\theta_2)$ holds.

Then, using $J(\tilde{\beta}|q)$ defined by (13),

$$\begin{aligned}h(\bar{\beta}^*|\underline{q}^*(\theta_2),\theta_1)-h(\bar{\beta}^*|\underline{q}^*(\theta_2),\theta_2)&=-\min\left\{-J(\bar{\beta}^*|\underline{q}^*(\theta_2))\middle|G\in\mathrm{core}(\theta_1)\right\}+\min\left\{-J(\bar{\beta}^*|\underline{q}^*(\theta_2))\middle|G\in\mathrm{core}(\theta_2)\right\}\\&=\max\left\{J(\bar{\beta}^*|\underline{q}^*(\theta_2))\middle|G\in\mathrm{core}(\theta_1)\right\}-\max\left\{J(\bar{\beta}^*|\underline{q}^*(\theta_2))\middle|G\in\mathrm{core}(\theta_2)\right\}\geq0\end{aligned}$$

The last inequality comes from the fact that $\mathrm{core}(\theta_1)\supseteq\mathrm{core}(\theta_2)$ and $J(\bar{\beta}^*|\underline{q}^*(\theta_2))>0$.

Therefore, $\underline{q}^*(\theta_1)\geq\underline{q}^*(\theta_2)$. ■

B.17 Proof of Proposition 4

As in the proof of Proposition 2, it suffices to prove that $h(\bar{\beta}^*|q,G_1)<h(\bar{\beta}^*|q,G_2)$ holds for any $q\in[\underline{q},1]$.

Denote

$$\begin{aligned}H(G,r)=&\int_0^{\bar{\beta}^*}(\beta^r-\beta)dG+\int_{\bar{\beta}^*}^{\min\left\{\bar{\beta}^*+\frac{q\delta\rho}{1-(1-q)\delta},\bar{\beta}\right\}}((\bar{\beta}^*)^r-\bar{\beta}^*)dG+\int_{\min\left\{\bar{\beta}^*+\frac{q\delta\rho}{1-(1-q)\delta},\bar{\beta}\right\}}^{\min\left\{\bar{\beta}^*+\left(\frac{q\delta\rho}{1-(1-q)\delta}\right)^{\frac{1}{r}},\bar{\beta}\right\}}((\bar{\beta}^*)^r-\beta)dG\\&+\int_{\min\left\{\bar{\beta}^*+\left(\frac{q\delta\rho}{1-(1-q)\delta}\right)^{\frac{1}{r}},\bar{\beta}\right\}}^{\bar{\beta}}(\beta^r-\beta)dG+\frac{\delta q}{1-\delta}\int_0^{\bar{\beta}^*}(\beta^r-\beta)dG+\frac{\delta q}{1-\delta}\int_{\bar{\beta}^*}^{\bar{\beta}}((\bar{\beta}^*)^r-\bar{\beta}^*)dG.\end{aligned}$$

Then,

$$h(\bar{\beta}^*|q,G_1)-h(\bar{\beta}^*|q,G_2)=(19)+H(G_1,r)-H(G_2,r). \tag{25}$$

In the above, $h(\bar{\beta}^*|q,G_1)-h(\bar{\beta}^*|q,G_2)$ is approximated by (19). As a matter of fact, $h(\bar{\beta}^*|q,G_1)-h(\bar{\beta}^*|q,G_2)=(19)$ when $r=1$. Here, $H(G_1,r)-H(G_2,r)$ represents the approximation error.

From the assumption, (19)<0 is negative, and (19) is independent of r. On the other hand, $H(G_1,r)-H(G_2,r)$ is continuous with respect to r, and zero when $r=1$. Therefore, there is $\bar{r}>1$ such that for any $r\in(1,\bar{r})$, the right-hand side of equation (25) is negative. ■